POPULATION-TEMPORAL CODING

Note: Projects...

Putting it all together

- Temporal repn (spikes) and population repn (distributed activities) have been independently considered
- Both are nonlinear encoding, linear decoding.
- Stick them together. Encoding:

$$\begin{aligned} a_i(\mathbf{x}(t)) &= G_i[J_i(\mathbf{x}(t))] \\ J_i(\mathbf{x}(t)) &= \alpha_i \left\langle \tilde{\phi}_i \mathbf{x}(t) \right\rangle_m + J_i^{bias}. \end{aligned}$$

- Decoding:

$$\hat{\mathbf{x}}(t) = \sum_{i,n} \phi_i h(t - t_{in})$$

PT Filtering

- Finding optimal h(t) as before implicitly includes the population decoder.
- So, always normalize h(t) (optimal or not) to area = 1 before using the decoders.
- So, decoders are, more accurately

$$\hat{\mathbf{x}}(t) = \sum_{i,n} \phi_i(t - t_{in})$$

Ideal PT decoder

- Should add noise to optimize

$$\hat{\mathbf{x}}(t) = \sum_{i,n} \phi_i(t - t_{in} - \eta_{in})$$

- Minimize

$$E = \left\langle \left[\mathbf{x}(t; \mathbf{A}) - \sum_{i,n} \phi_i(t - t_{in} - \eta_{in}) \right]^2 \right\rangle_{\mathbf{A},\eta}$$

- Technically should use a Monte Carlo method

Considered independently

- we can then use a non-optimal, biologically plausibly temporal decoder (the PSC)
- we can find the decoders analytically
- we can easily apply other (as yet unseen) analyses which help us understand population representation

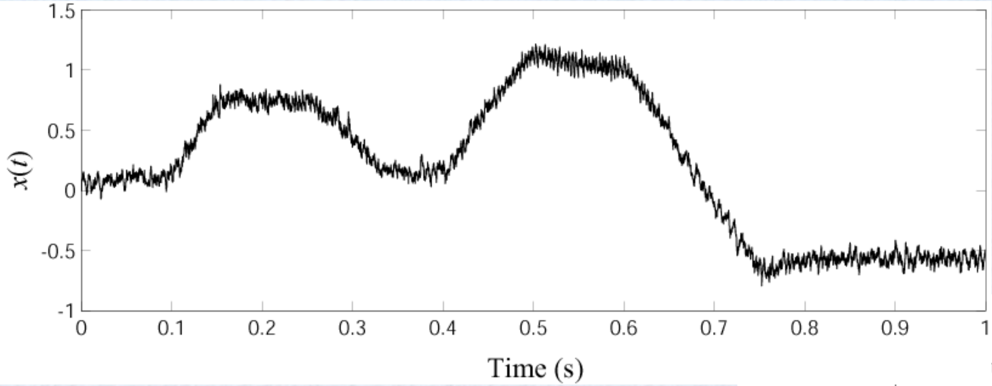
Why should it work?

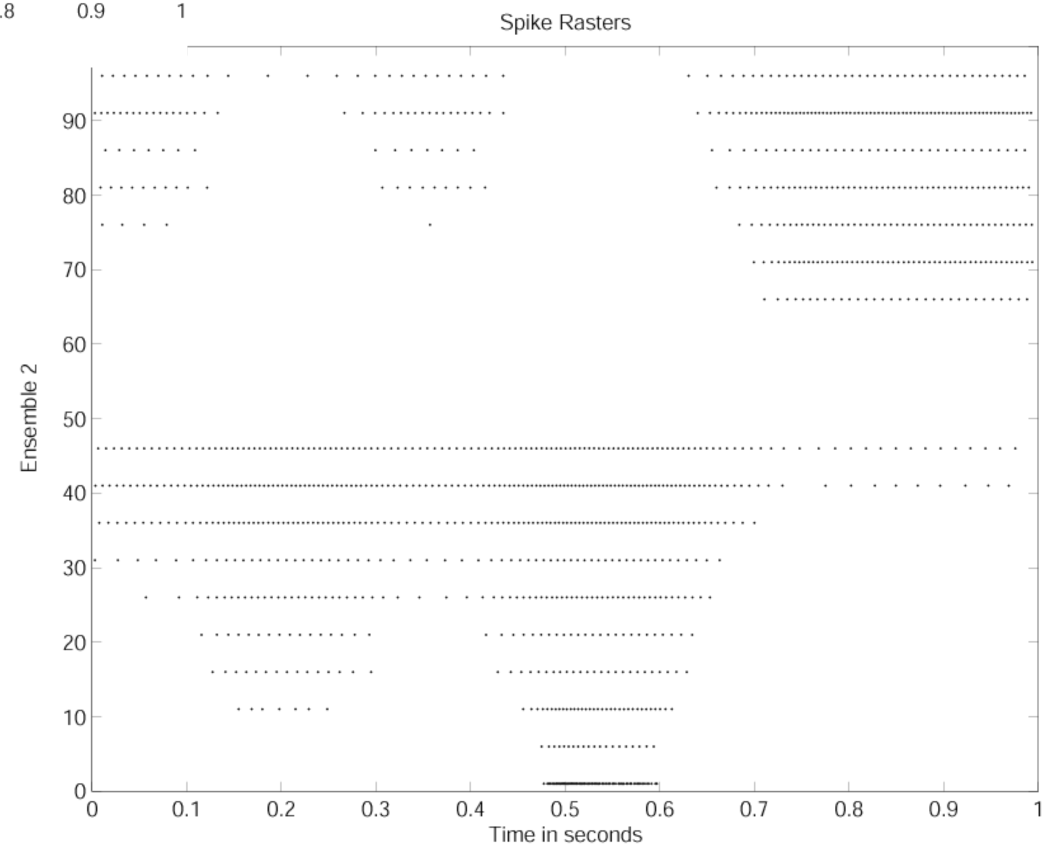
- The decoders are the same because we used a LIF in both cases, so

$$
\begin{aligned}
a_i^{rate}(\mathbf{x}) &= \left\langle \sum_n h_i(t) * \delta(t - t_{in}) \right\rangle_T \\
&= \left\langle \sum_n h_i(t - t_{in}) \right\rangle_T \\
&= \left\langle a_i^{spiking}(\mathbf{x}) \right\rangle_T
\end{aligned}
$$

- Will be convincing if we can build models well

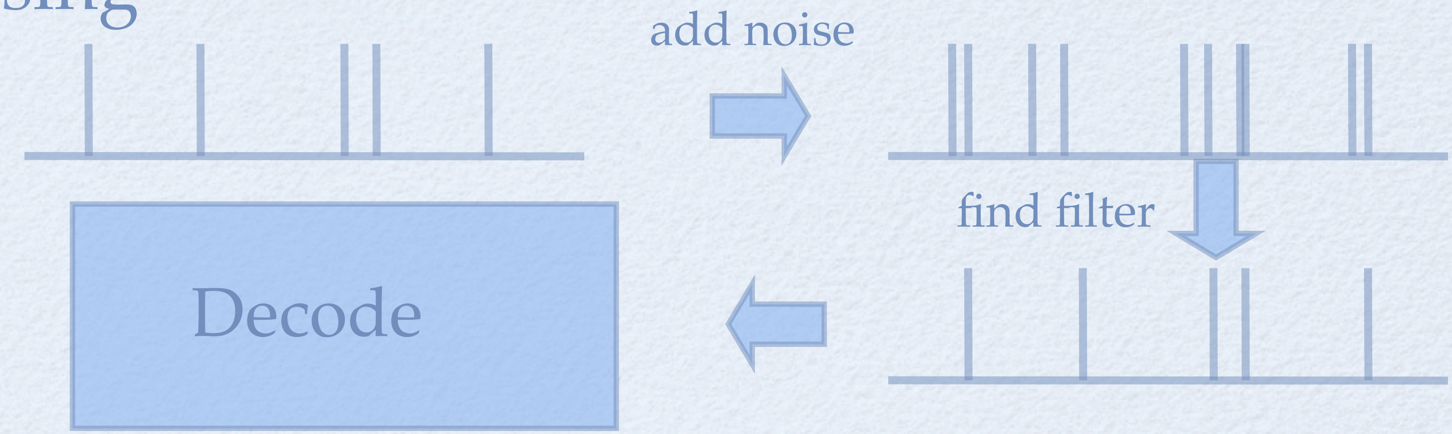
Population-temporal filter



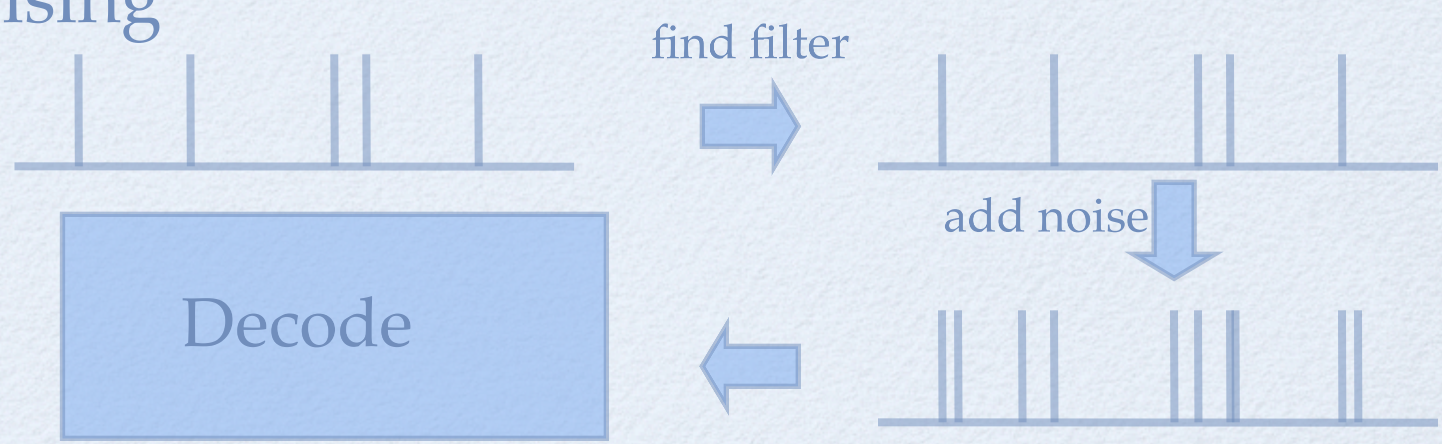


Noise and precision

- Pre-noising



- Post-noising



Fluctuations as noise

- Using spikes results in fluctuations in the estimate that are like more noise (so why spike?)

- Appendix C.1 has details. Postsynaptic activity, under contant input:

$$\alpha_i(x,t) = \sum_n h_i\left(t - n\Delta_i(x) - t_{i_0}\right)$$

- Variance is:

$$\sigma^2_{\hat{x}(t)} = \left\langle \left[\hat{x}(t) - \langle \hat{x}(t)\rangle_T\right]^2 \right\rangle_{T,t_{i0}}$$

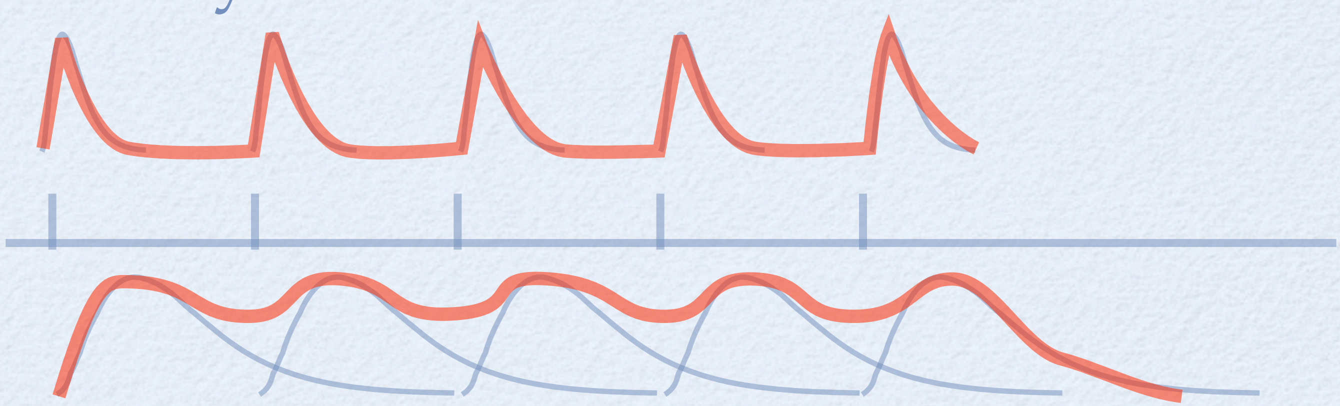
Fluctuations

- Variance becomes

$$\sigma^2_{\hat{x}(t)} = \sum_i \phi_i^2 a_i(x) \left[\sum_m g_i\left(m\Delta_i(x)\right) - a_i(x) \right]$$

$$g_i(\tau) = \int_{-\infty}^{\infty} h_i(t) h_i(t-\tau) dt$$

- Intuitively as τ increases:



Error

- Error comes from 3 sources

$$
\begin{aligned}
E_{total} &= E_{static} + E_{noise} + E_{fluctuations} \\
&= \frac{1}{2}\left\langle \left[x - \sum_i a_i(x)\phi_i \right]^2 \right\rangle_x + \sigma_\eta^2 \sum_i \phi_i^2 + \sigma_{\hat{x}(t)}^2
\end{aligned}
$$

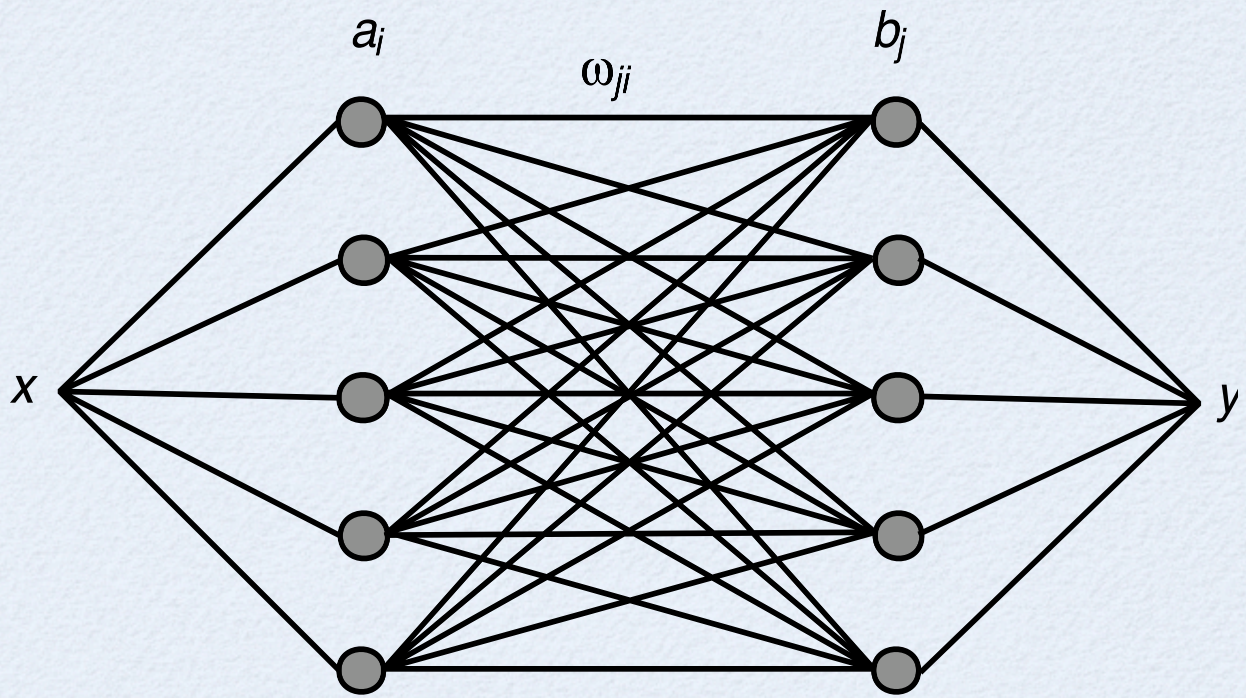
- Because the last two are the same form, they can be combined into

$$\sigma^2 \sum_i \phi_i^2$$

- The previous noise analysis will work here (1/N)

Feedforward Transformations

A communication channel



Connection weights

- Define the representations of both pops:

$$a_i(x) = G_i\left[J_i(x)\right] = G_i\left[\alpha_i \tilde{\phi}_i x + J_i^{bias}\right]$$

$$\hat{x} = \sum_i a_i(x)\phi_i^x$$

$$b_j(y) = G_j\left[J_j(y)\right] = G_j\left[\alpha_j \tilde{\phi}_j y + J_j^{bias}\right]$$

$$\hat{y} = \sum_j b_j(y)\phi_j^y$$

- Define the computation: $y = x$
- Substitute our estimate of x into b

Connection weights

- Substituting: $y = x \approx \hat{x}$

$$
\begin{aligned}
b_j(x) &= G_j\left[\alpha_j \tilde{\phi}_j x + J_j^{bias}\right] \\
&= G_j\left[\alpha_j \tilde{\phi}_j \sum_i a_i(x)\phi_i^x + J_j^{bias}\right] \\
&= G_j\left[\sum_i \omega_{ji} a_i(x) + J_j^{bias}\right]
\end{aligned}
$$

$$\omega_{ji} = \alpha_j \tilde{\phi}_j \phi_i^x$$

With spikes

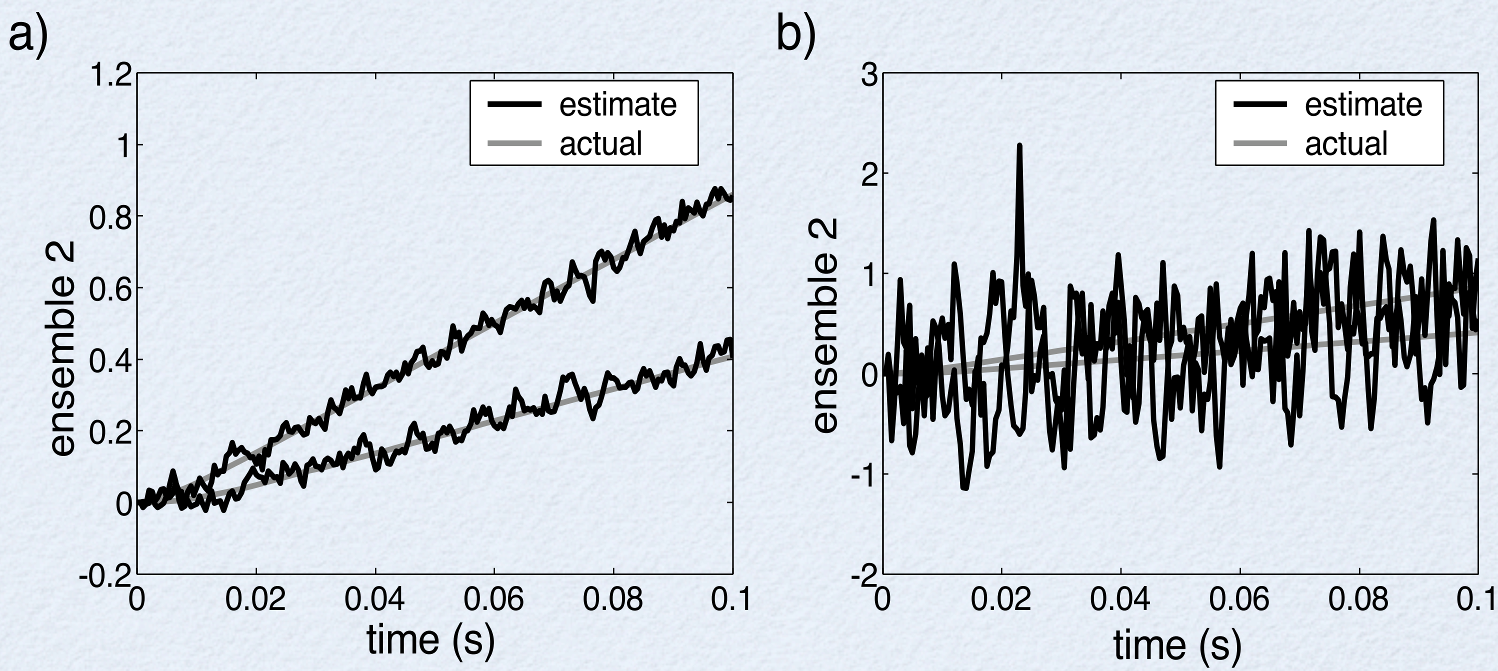
- Write the spiking estimate $$\hat{x}(t) = \sum_i a_i(x(t))\phi_i^x = \sum_{i,n} h_i(t - t_{in})\phi_i^x$$
- Then do the same substitution:

$$\begin{aligned} b_j(x(t)) &= G_j\left[\alpha_j\tilde{\phi}_j x(t) + J_j^{bias}\right] \\ &= G_j\left[\alpha_j\tilde{\phi}_j \sum_{i,n} h_i(t - t_{in})\phi_i^x + J_j^{bias}\right] \\ &= G_j\left[\sum_{i,n} \omega_{ji} h_i(t - t_{in}) + J_j^{bias}\right] \end{aligned}$$

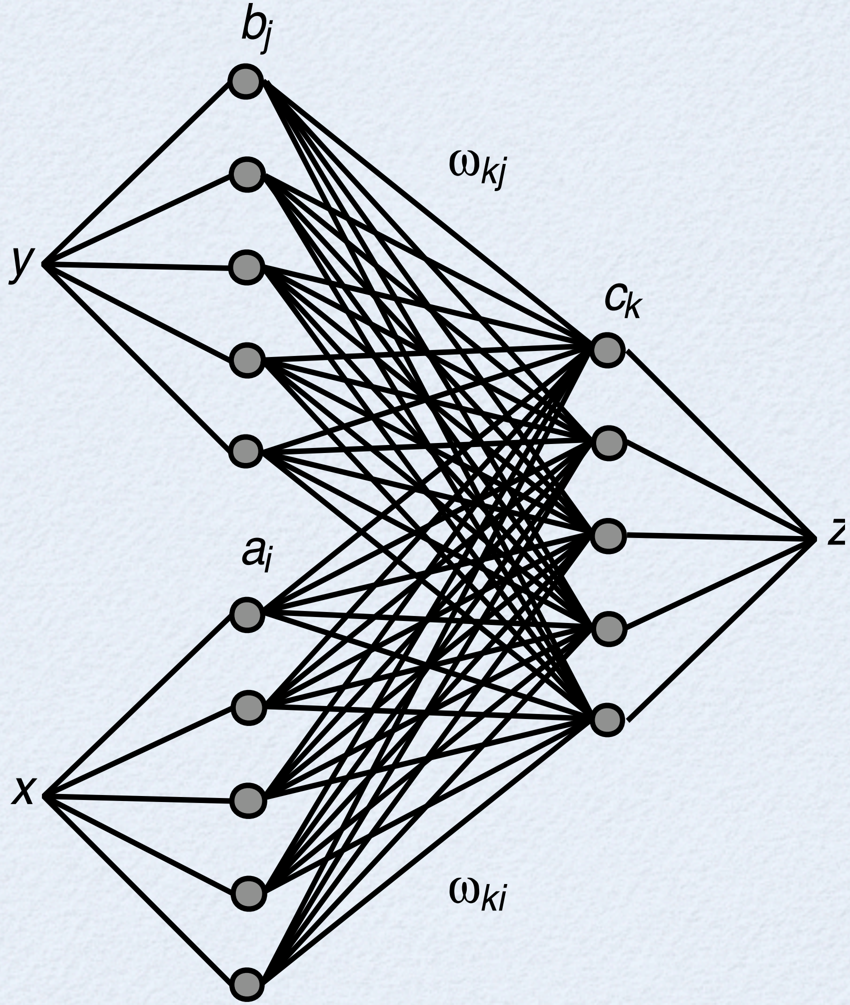
Scaling and noise

- x and $2x$

  In b) the decoders weren't found under noise



Adding scalars



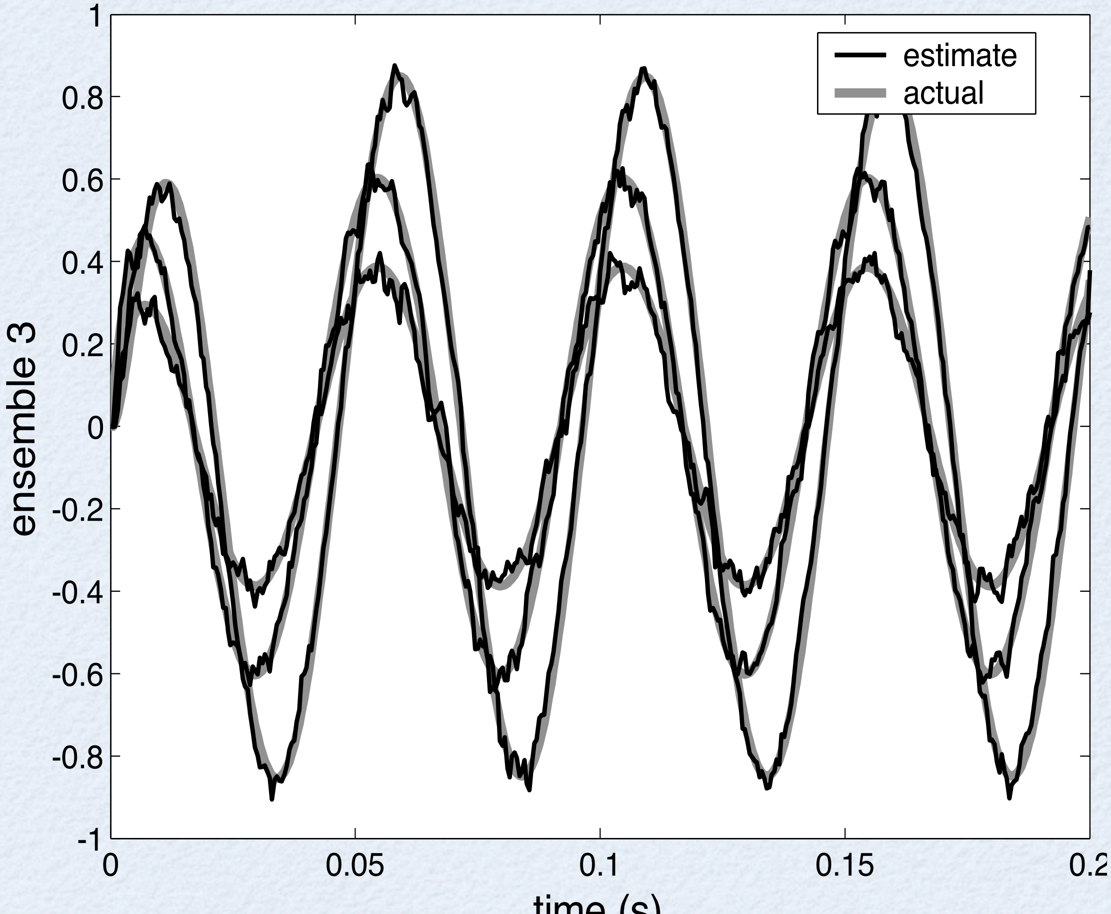
Doing the math

- Volunteer?

$$
\begin{aligned}
c_k(x+y) &= G_k\left[\alpha_k \tilde{\phi}_k(x+y) + J_k^{bias}\right] \\
&= G_k\left[\alpha_k \tilde{\phi}_k \left(\sum_i a_i(x)\phi_i^x + \sum_j b_j(y)\phi_j^y\right) + J_k^{bias}\right] \\
&= G_k\left[\sum_i \omega_{ki} a_i(x) + \sum_j \omega_{kj} b_j(y) + J_k^{bias}\right]
\end{aligned}
$$

$$
\omega_{ki} = \alpha_k \tilde{\phi}_k \phi_i^x \quad \omega_{kj} = \alpha_k \tilde{\phi}_k \phi_j^y
$$

Scalar addition



Recipe for linear trans.

- 1. Define the repn (enc/dec) for all variables involved in the operation.
- 2. Write the transformation in terms of these variables.
- 3. Write the transformation using the decoding expressions for all variables except the output variable.
- 4. Substitute this expression into the encoding expression of the output variable.

Vectors

- Nothing new. Representation:

$$\begin{aligned} a_i(\mathbf{x}) &= G_i\left[\alpha_i\left\langle\tilde{\phi}_i\mathbf{x}\right\rangle_m + J_i^{bias}\right] \\ \hat{\mathbf{x}} &= \sum_i a_i(\mathbf{x})\phi_i^{\mathbf{x}} \end{aligned}$$

- Transformation

$$\mathbf{z} = C_1\mathbf{x} + C_2\mathbf{y}$$

Vectors

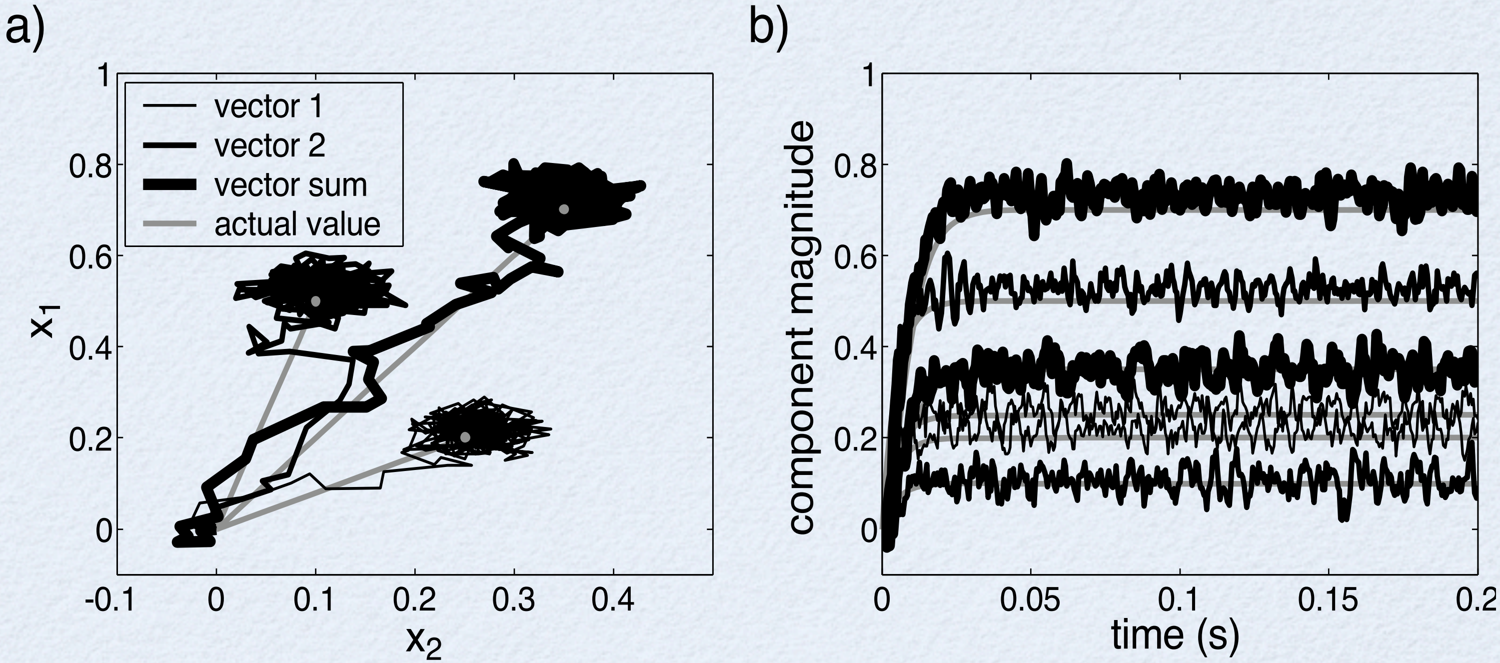
- Substitution

$$
\begin{aligned}
c_k(C_1\mathbf{x}+C_2\mathbf{y}) &= G_k\left[\alpha_k\left\langle\tilde{\phi}_k(C_1\mathbf{x}+C_2\mathbf{y})\right\rangle_m + J_k^{bias}\right] \\
&= G_k\left[\alpha_k\left\langle\tilde{\phi}_k\left(C_1\sum_i a_i(\mathbf{x})\phi_i^{\mathbf{x}} + C_2\sum_j b_j(\mathbf{y})\phi_j^{\mathbf{y}}\right)\right\rangle_m + J_k^{bias}\right] \\
&= G_k\left[\sum_i \omega_{ki}a_i(\mathbf{x}) + \sum_j \omega_{kj}b_j(\mathbf{y}) + J_k^{bias}\right]
\end{aligned}
$$

$$
\omega_{ki} = \alpha_k C_1\left\langle\tilde{\phi}_k\phi_i^{\mathbf{x}}\right\rangle_m \quad \omega_{kj} = \alpha_k C_2\left\langle\tilde{\phi}_k\phi_j^{\mathbf{y}}\right\rangle_m
$$

Vector addition

- a) vector space; b) components



Comments

- Use matrices instead of scalars: $\omega_{ki} = \alpha_k \left\langle \tilde{\phi}_k \mathbf{C}_1 \phi_i^{\mathbf{x}} \right\rangle_m$
- Permits any linear operation (rotation, scaling)
- Spiking neurons: $a_i(\mathbf{x}) = \sum_n h(t - t_{in})$
- Does a good job of vector addition (small transient with spikes because of).
- Improve performance by adding neurons
- This kind of network may be used in frontal eye fields for control of saccades.