# Admin

- Posted assignment due dates:
  - [2 (2 weeks) 20 marks] due: Jan 24
  - [3 (2 weeks; see hint below) 20 marks] due: Feb 16
  - [5 (2 weeks) 10 marks] due: Mar 2
  - [8 (1 week) 10 marks] due: Mar 16
- Spafford tour March 9 - 11:00a, B1-172

# Representation

- Lossless code:
  - Encoding: $a = f(x)$
  - Decoding: $x = f^{-1}(a)$
- Otherwise:
  - Decoding: $\hat{x} = g(a) \approx f^{-1}(a)$

# Engineered Representation

- A/D converter (x is an input voltage)
- Encode $a_i(x) = \begin{cases} 1, & \text{if } x \bmod 2^i \geq 2^{i-1} \\ 0, & \text{otherwise} \end{cases}$
    - ($x$=$x$-$2^{i-1}$ after each 1)
- Decode $\hat{x} = \sum_{i=1}^{N} a_i(x)\phi_i$

where $\phi_i = 2^{i-1}$

# Binary encode/decode

- e.g. $x = 10$
- 10 mod $2^4 = 10 > 2^3$ -> 1 ($x = 10-2^3$)
- 2 mod $2^3 = 2 < 2^2$ -> 0
- 2 mod $2^2 = 2 >= 2^1$ -> 1 ($x = 2-2^1$)
- 0 mod $2^1 = 0 < 2^0$ -> 0
- $x_{est} = 1*(2^3) + 0*(2^2) + 1*(2^1) + 0*(2^0) = 8+0+2+0$

# Important lessons

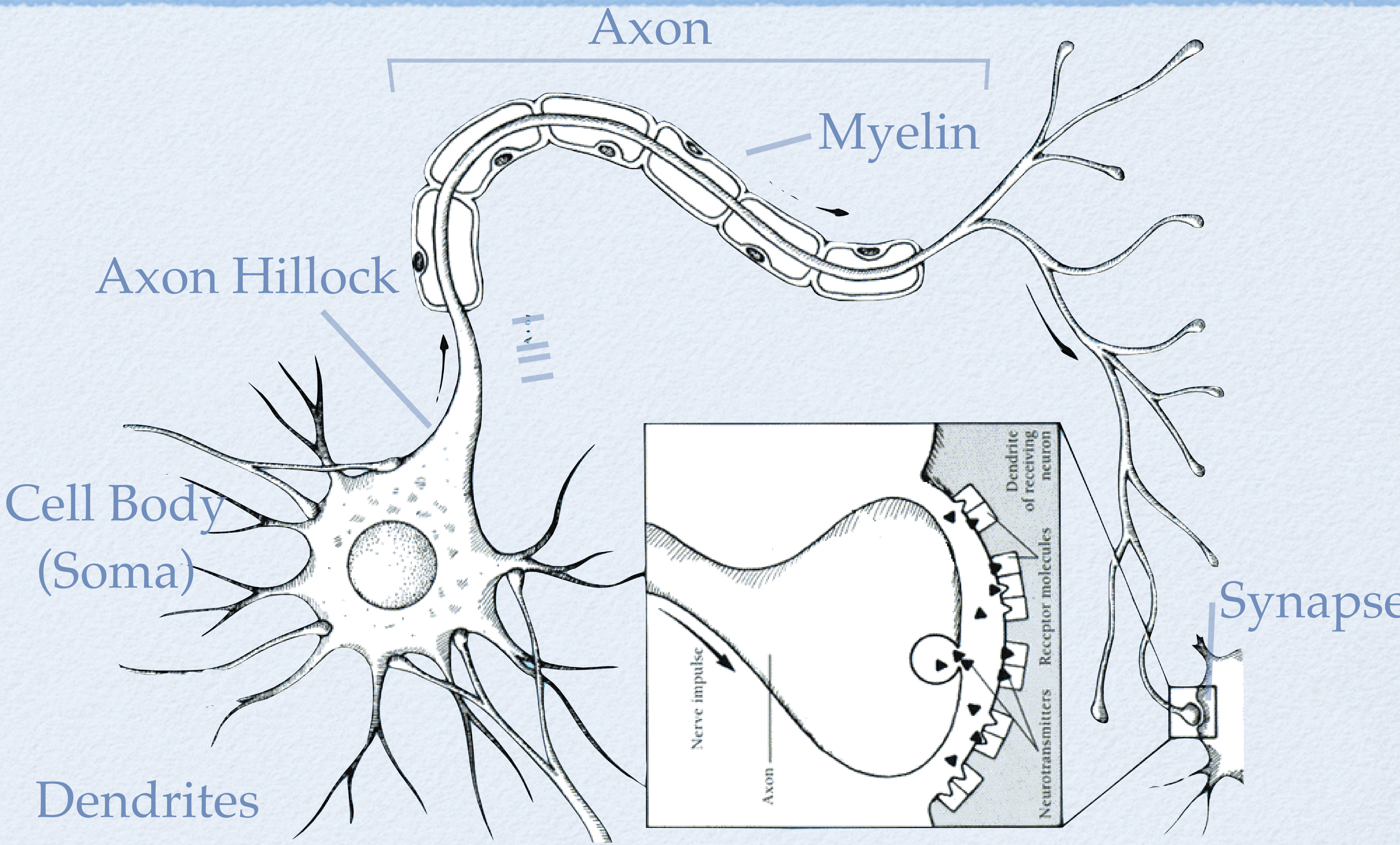
- Nonlinear encoding can be decoded linearly
- Encoding and decoding must be defined together
- Aside: distributed represention is implementation relative

# Biological Representation

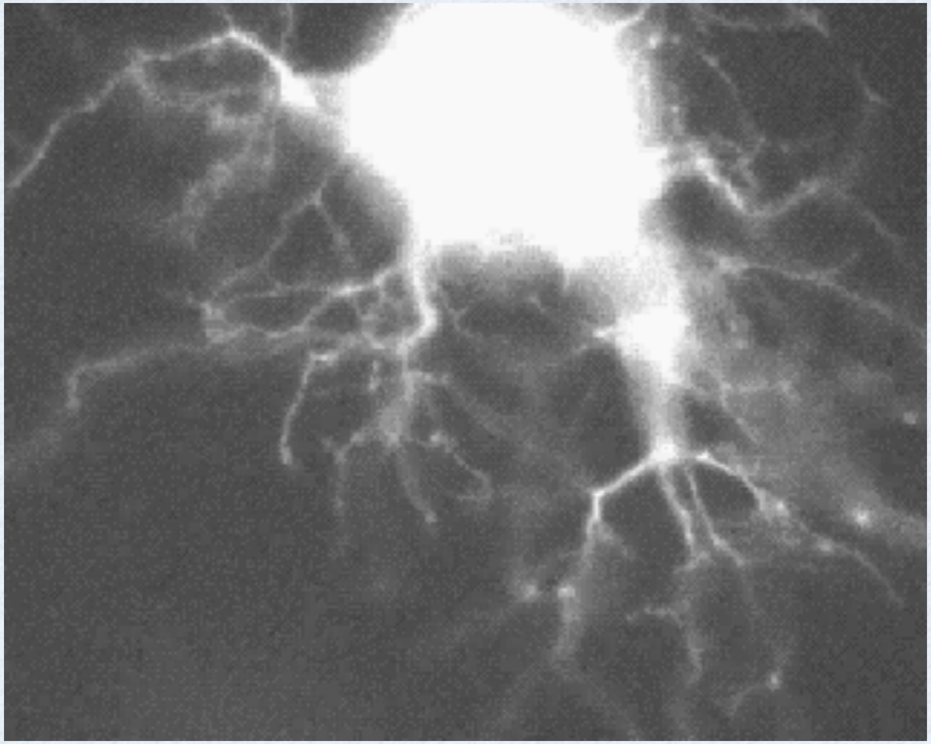
- Experimentally, we know the encoding is of the form:

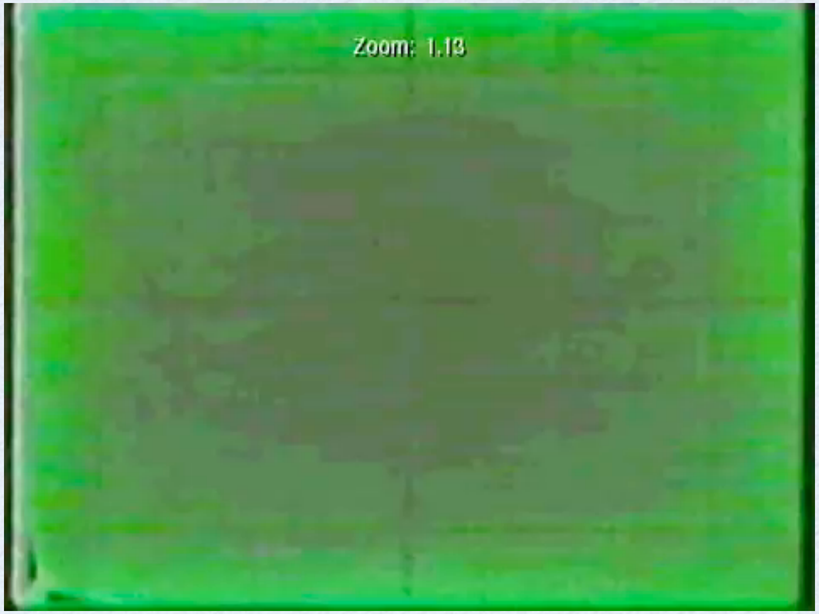
$$a_i(J) = G_i\left[J\right]$$

# A single neuron



# Nonlinear encoding





# Nonlinear encoding

- More specifically, we know:

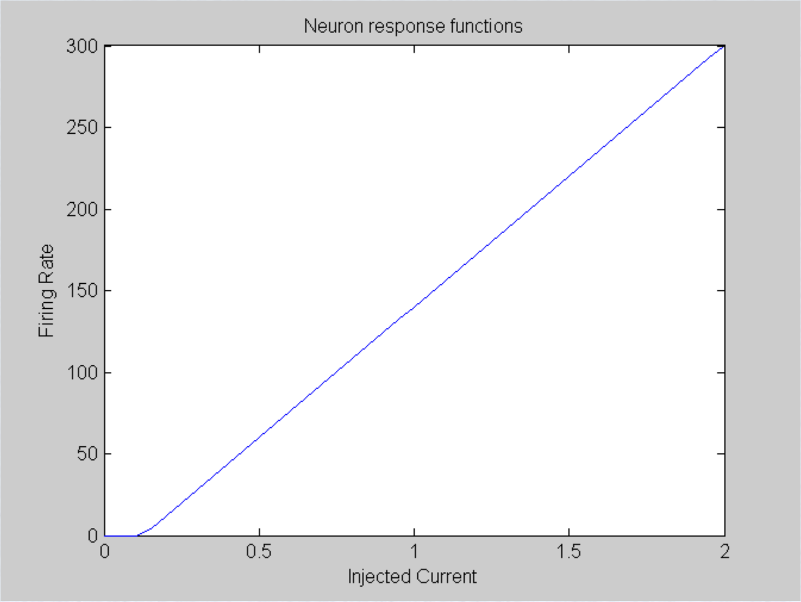
$$J(x) = \alpha x + J^{bias}$$

- So

$$a_i(x) = G_i\left[\alpha x + J^{bias}\right]$$

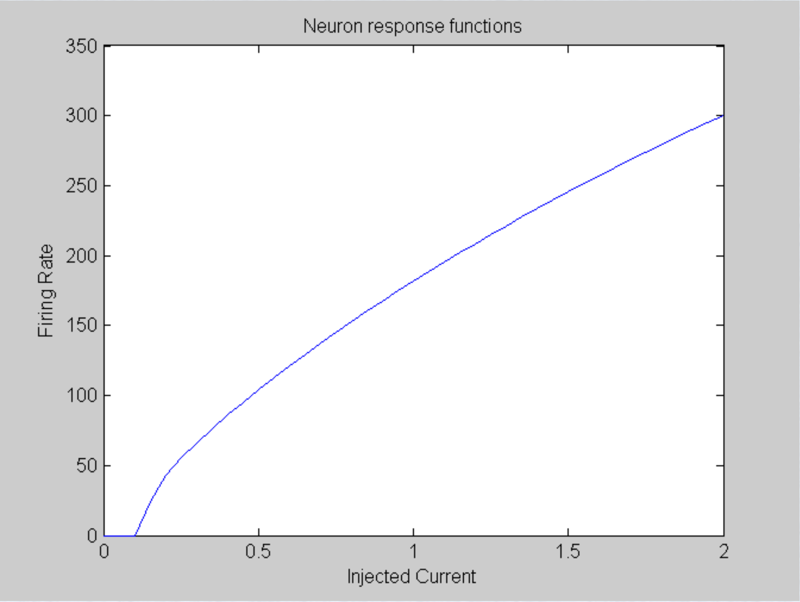
# Model response functions

## Rectified Linear




$$a(x) = [J(x)]_+$$

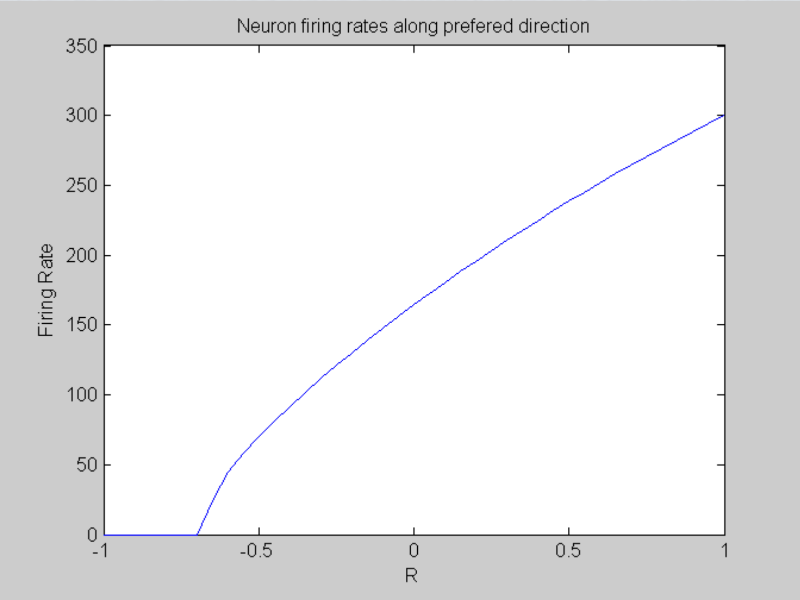
## LIF Neuron



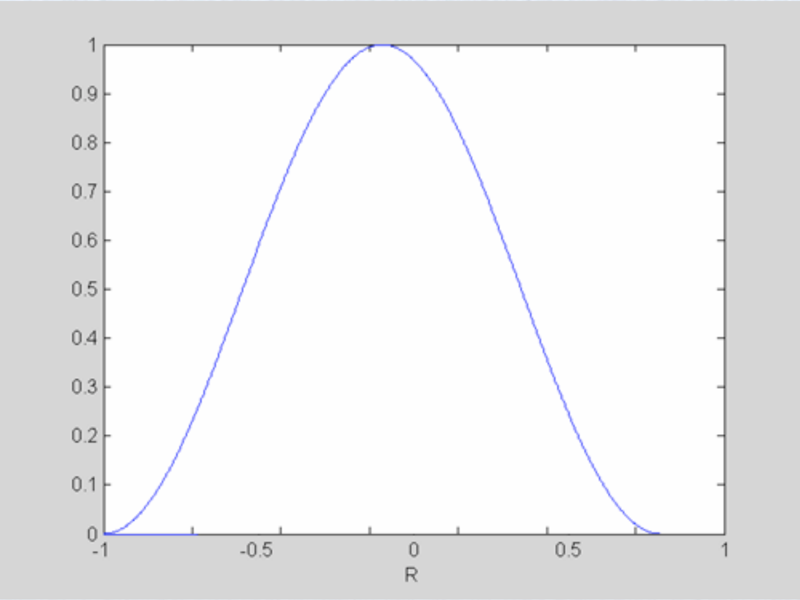

$$a(x) = \frac{1}{\tau_i^{ref} - \tau_i^{RC} \ln\left(1 - \frac{J^{threshold}}{J(x)}\right)}$$

# Tuning curves

## Motor neuron




## Perceptual neuron




$$a_i(x) = \frac{1}{\tau_i^{ref} - \tau_i^{RC} \ln\left(1 - \frac{J_i^{threshold}}{\alpha_i x + J^{bias}}\right)}$$

# Decoding

- Linear:

$$\hat{x} = \sum_i a_i(x)\phi_i$$

- Note: Must use tuning curves
- Q: How to find decoders?
- A: Minimize $\left\langle (x - \hat{x})^2 \right\rangle_x$ (see notes)

# Differences

(From engineering representations)

- Result in analog quantities
- Need to have the decoders determined in order for us to analyze them as repn.
- Are 'more distributed,' meaning:
  - failure of an arbitrary neuron will affect the repn by about the same amount
  - the neuron encoding is highly redundant

# Similarities

- Have encoders that are similar but non-identical
- Are distributed
- Have increasing precision with more encoders
- Have nonlinear encoders and linear decoders

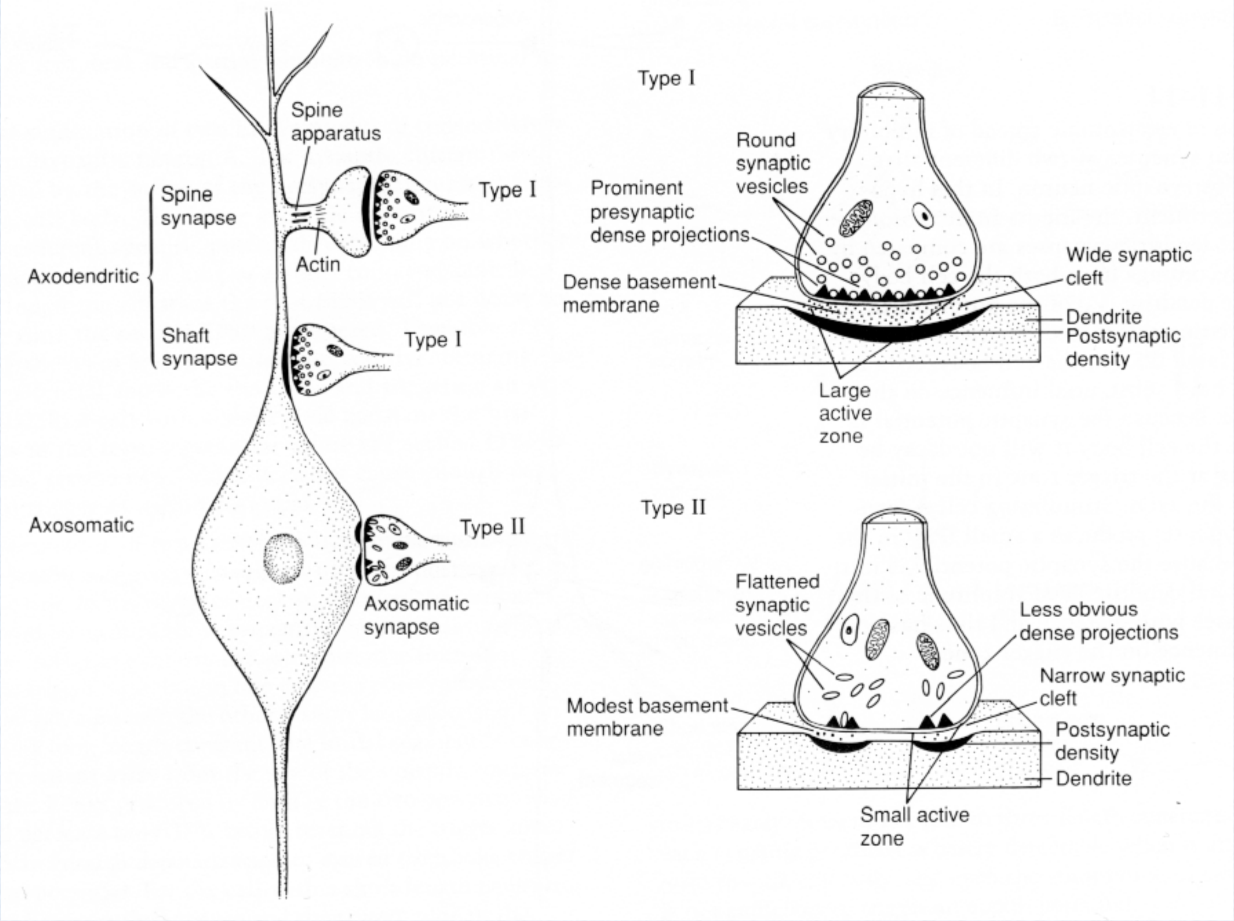
# Tutoring services

- Xuan Choo and Travis DeWolf
- $20/half hour or $15/person for groups
- ctntutoring@gmail.com

# Sources of Noise

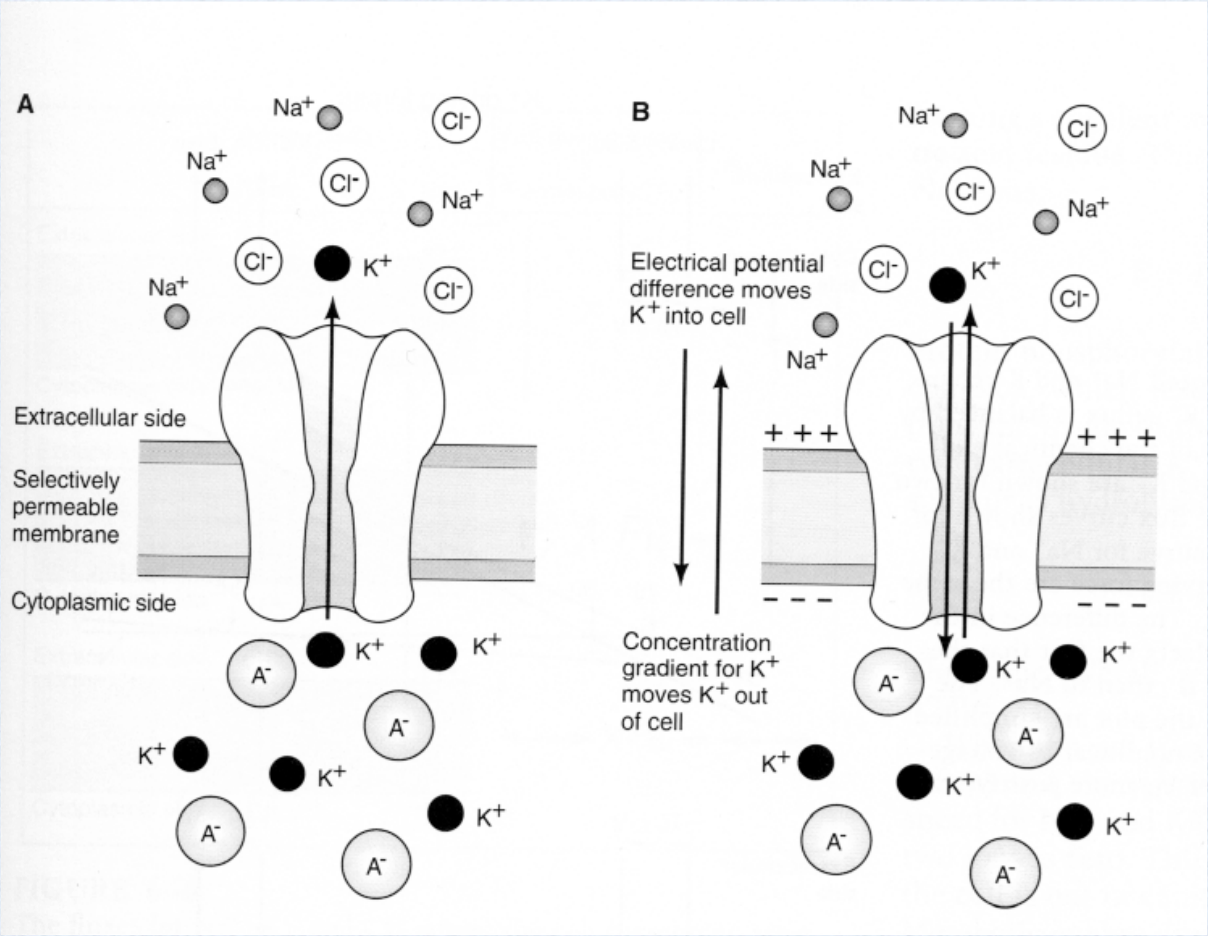
- Axonal jitter
- Neurotransmitter vesicle release failures (see next slide)
- Different amount of transmitter in each vesicle
- Thermal noise (minor)
- Ion channel noise (the number of channels open or closed fluctuates)
- Network effects.
- See also http://diwww.epfl.ch/~gerstner/SPNM/node33.html

# Synapses



Spine apparatus
Spine synapse
Axodendritic
Actin
Type I
Shaft synapse
Type I
Axosomatic
Type II
Axosomatic synapse
Type I
Round synaptic vesicles
Prominent presynaptic dense projections
Dense basement membrane
Wide synaptic cleft
Dendrite
Postsynaptic density
Large active zone
Type II
Flattened synaptic vesicles
Less obvious dense projections
Narrow synaptic cleft
Modest basement membrane
Postsynaptic density
Dendrite
Small active zone

# Ion channels



# Noise

- Must consider the decoding under noise:

$$\hat{x} = \sum_{i=1}^{N} \left(a_i(x) + \eta_i\right) \phi_i$$ (see notes)

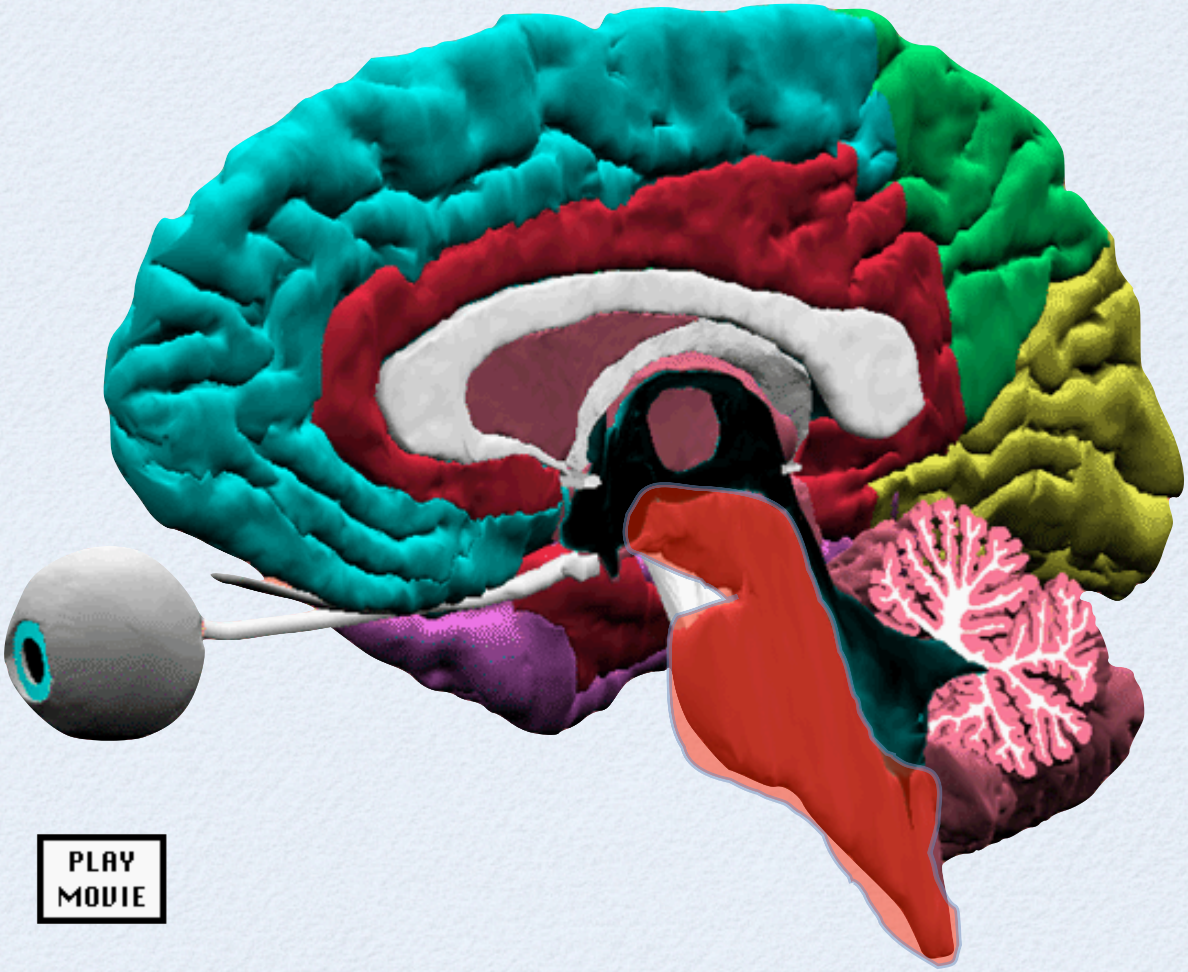
error due to static distortion — error due to noise

$$E = \frac{1}{2} \left\langle \left[ x - \sum_{i=1}^{N} a_i(x) \phi_i \right]^2 \right\rangle_x + \frac{1}{2} \sigma^2 \sum_{i=1}^{N} \phi_i^2 .$$

# Horizontal eye position

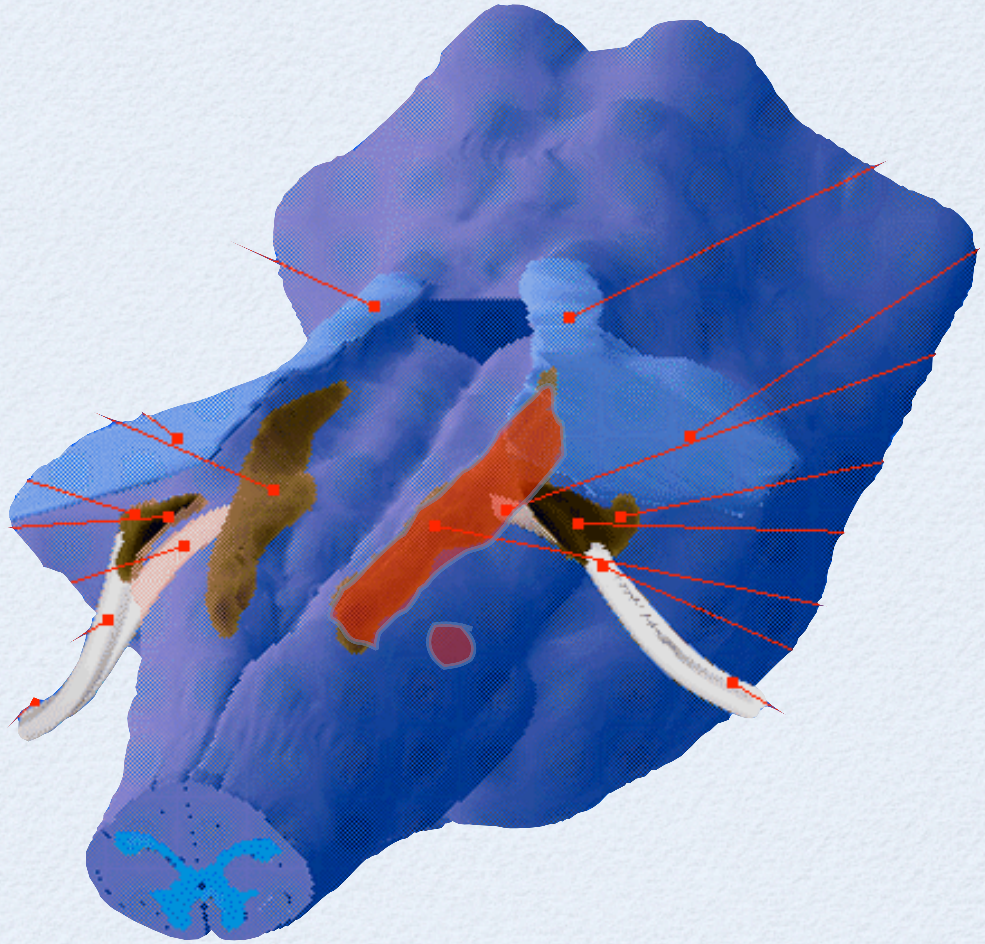
- System description is detailed in the book
- Neurons in these areas are very well-modeled by LIF neurons
- quick overview...

# Brain stem

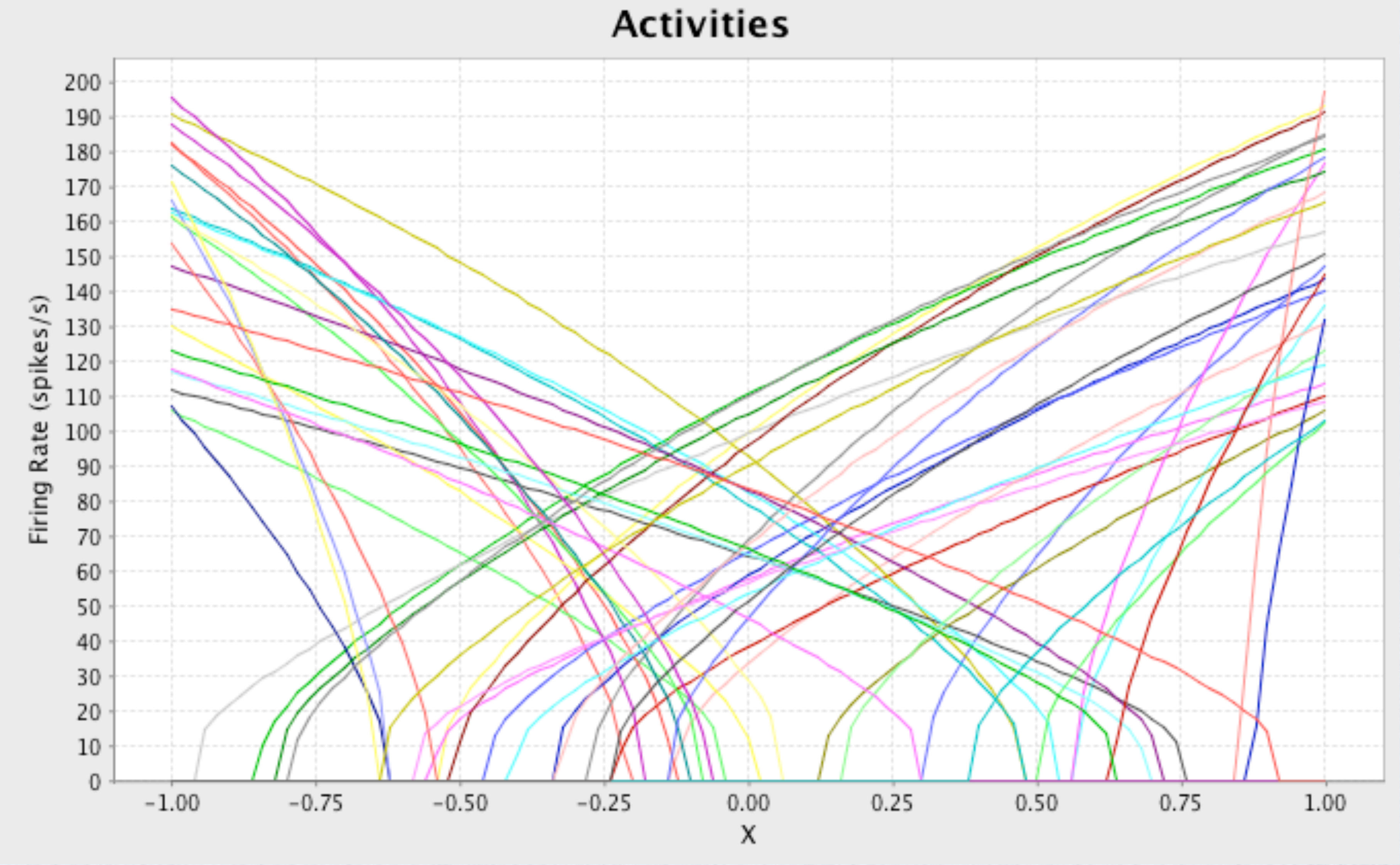


PLAY
MOVIE

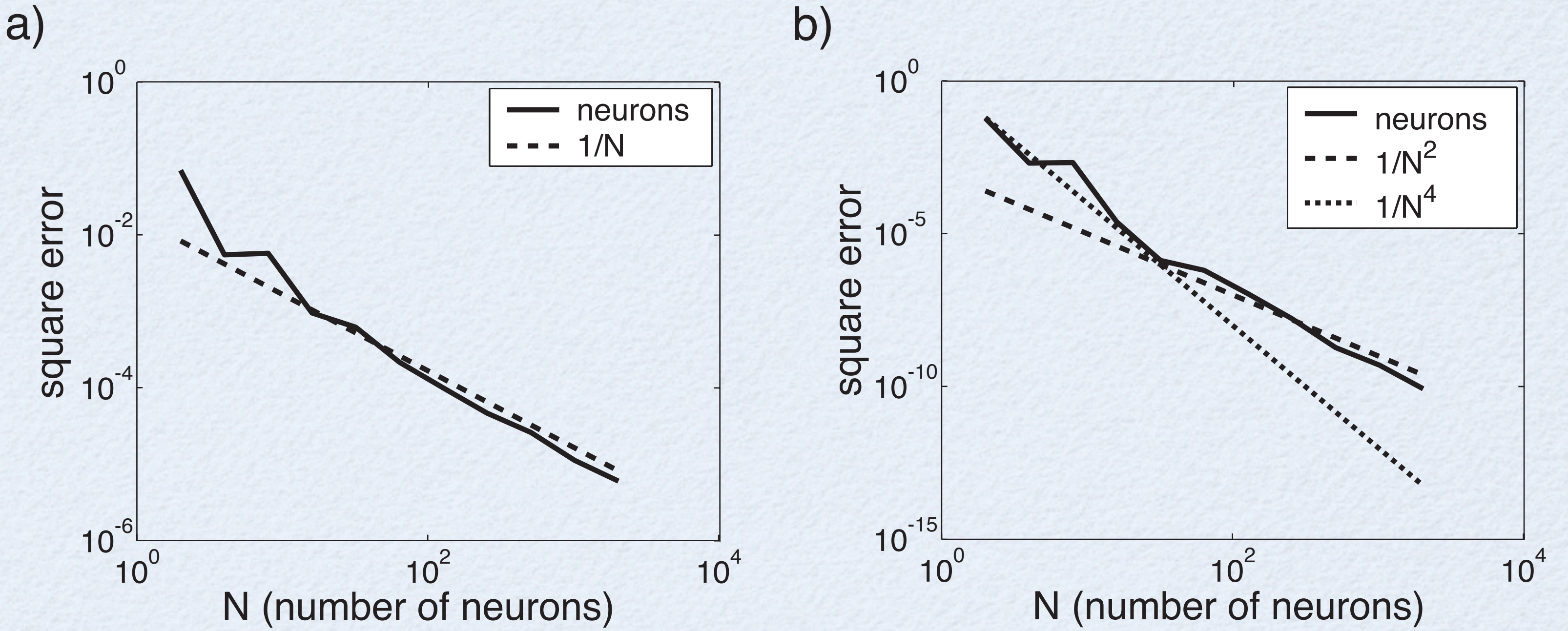
# NPH and Vestibular nuclei



# Population tuning



# Error with/without noise



# Vector representation

- Encoding:

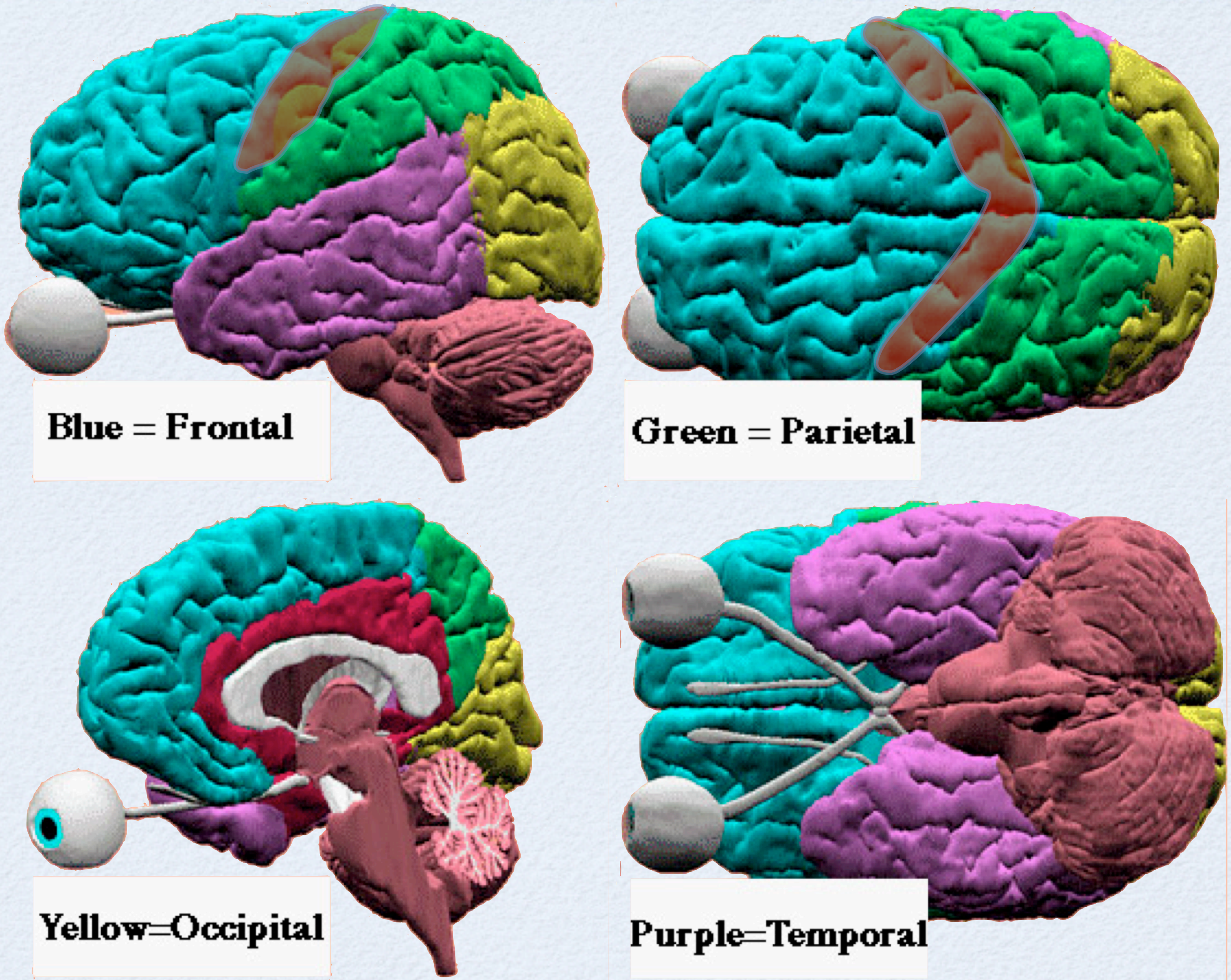
$$a_i(\mathbf{x}) = G_i\left[\alpha_i \left\langle \tilde{\boldsymbol{\phi}}_i \mathbf{x} \right\rangle_n + J_i^{bias}\right]$$

- Decoding

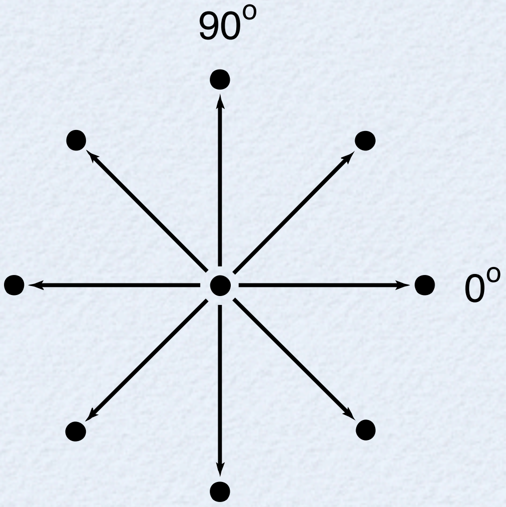
$$\hat{\mathbf{x}} = \sum_{i=1}^{N} a_i(\mathbf{x}) \phi_i$$

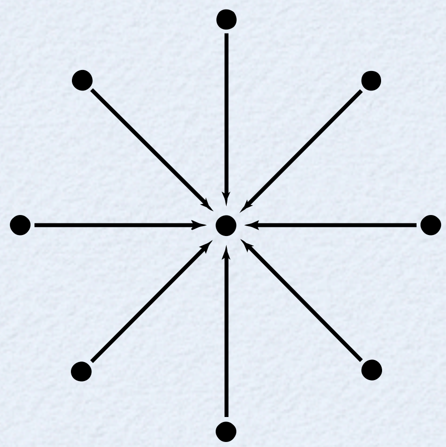
# Basic Brain Areas

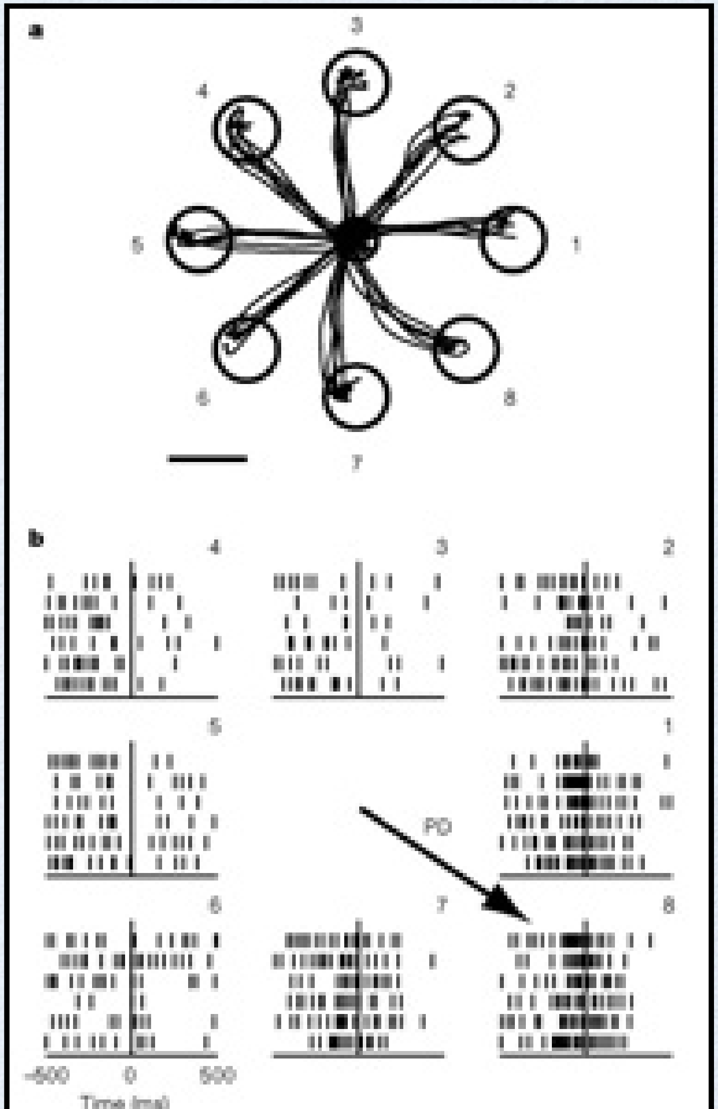
## primary motor cortex



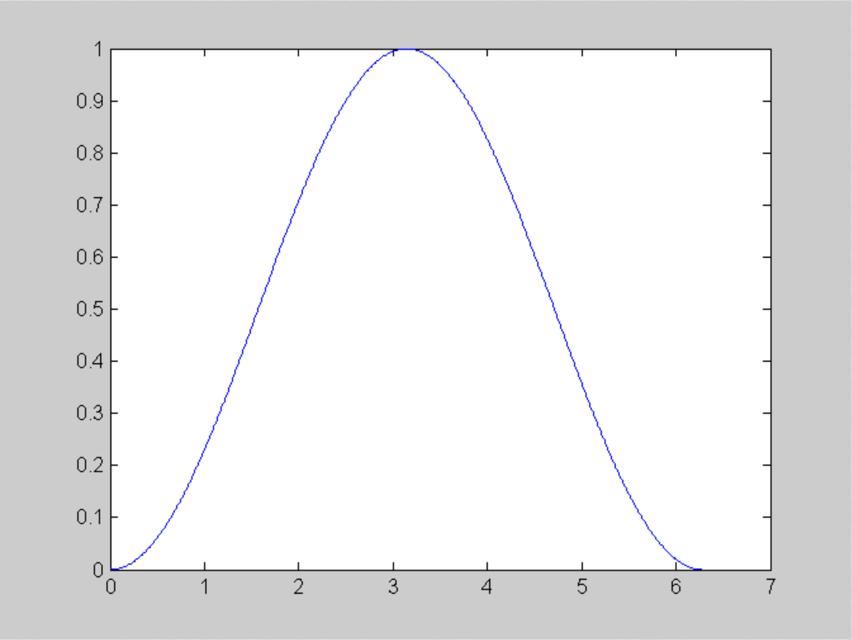
# Georgopolous task



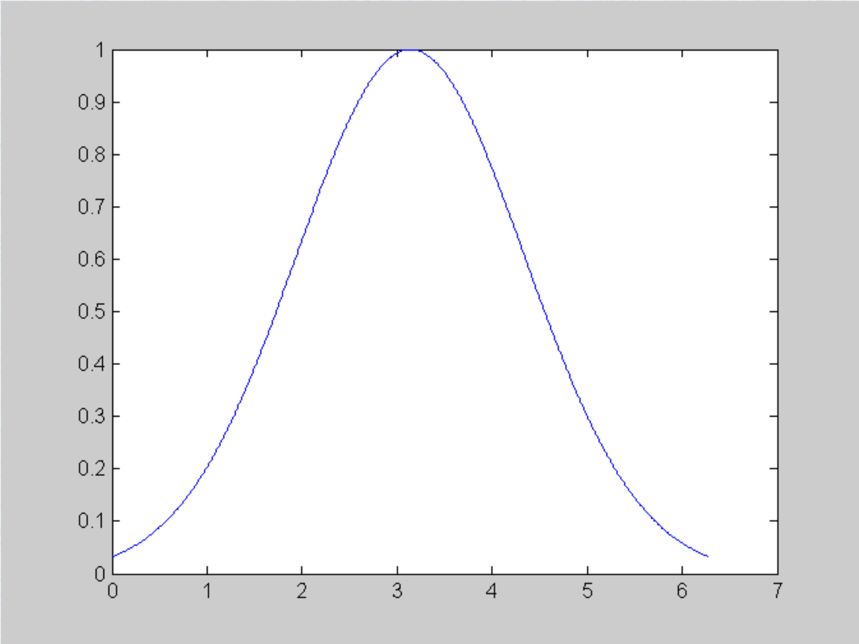





# Motor cortex tuning curves



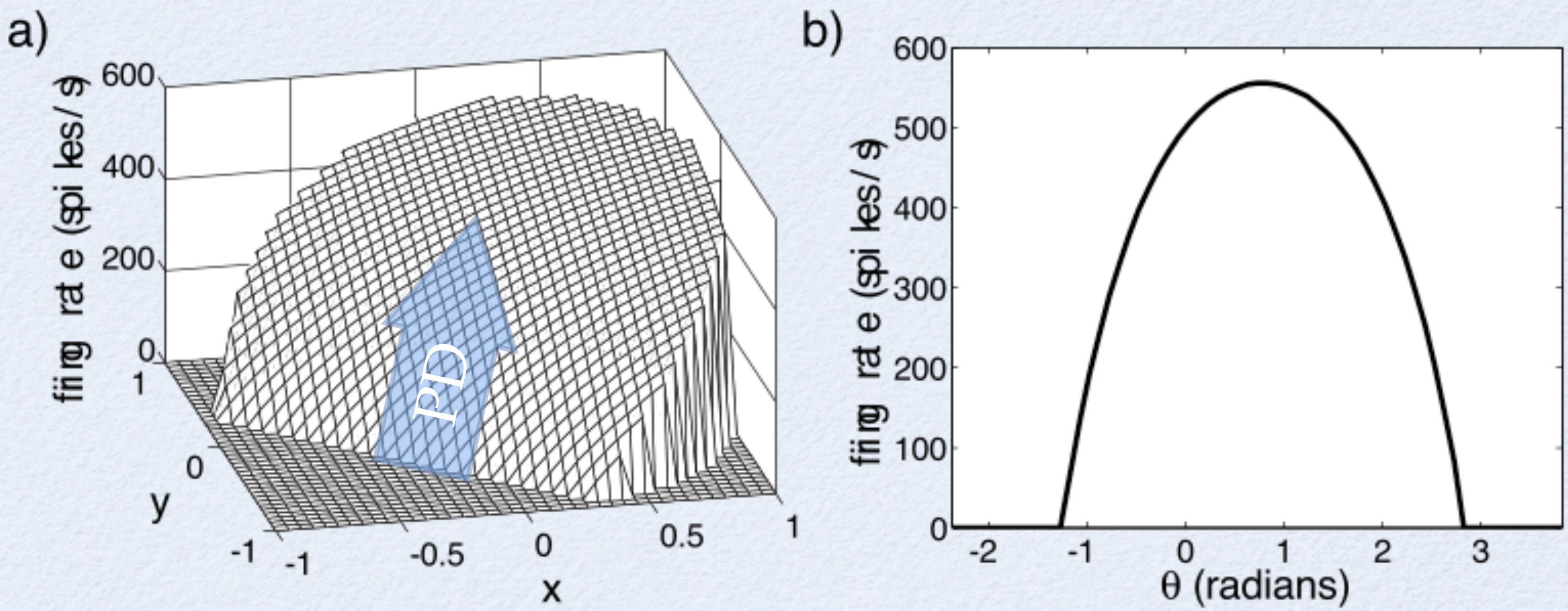

Cosine




Gaussian

# LIF 2D and cosine tuning

$$< \mathbf{x} \cdot \mathbf{y} >= |\mathbf{x}||\mathbf{y}|cos(\theta)$$



# Cosines and Decoding

- Cosine tuning is theoretically optimal for motor control in 3D (Flash and Sejnowski, 2001; Todorov, 2002)
- Georgopolous originally suggested using encoders as decoders, estimates angle but not magnitude.
- However, combined coding of direction and amplitude has been (more recently) shown (Fu et al., 1993; Messier and Kalaska, 2000)

# Motor and Vestibular

- Motor cortex has even distribution of encoding vectors
- Vestibular system has encoders only along a few axes

# Comparing preferred vectors

The system under study is important! (There are also significant computational differences)