Exercise 5: Temporal representation in spiking neurons

5.1 Population-temporal codes

1. Let $\tau^{RC} = 20$ ms, $\tau^{ref} = 2$ ms, $J_{th} = 1$, $\sigma = .1$ (normalized). Pick neurons from an even distribution over [-2,2] with maximum firing rates between 200-300Hz.

   Submit:

   (a) Population plot for N=20 LIF neurons.

   (b) Plot of noise-resistant optimal decoders (sorted by x-intercept of neurons).

   (c) $(x - \hat{x})$ with and without $a_i(x)$ subject to noise and with and without the determination of ϕ subjected to noise (i.e. 4 graphs).

2. Use PSCs ($\tau_{psc} = 5$ ms) to decode spikes from neurons in the previous population. Pick one neuron from the previous population, construct a mirror image, and decode their response to bandlimited white noise to 5 Hz ($dt = .001$, $rms = 1$, $T = 1$ s).

   Submit:

   (a) A plot of the PSC.

   (b) A plot of the decoding (original signal overlayed with the PSC decoding, and spikes).

   (c) The MSE of the decoding.

   Notes:

   - When you pick one neuron, pick one that has a background firing rate of 20-50Hz at zero, then use it and a mirror image of it to do the encoding and decoding.
   - When you decode, you should find the optimal decoder (over 4 seconds perhaps) so you can normalize the PSC to the same area.
   - I highly recommend doing the decoding as a sum of PSCs in the time domain instead of recycling code in the freq domain from the last assignment.
   - Always assume $n = 0$ for the PSC (i.e., $h(t) = e^{-t/\tau}$).

3. Combining the two above, examine the error as the number of neurons changes, N =2, 4, 8, 16, 32, 64, 128, ... more if you have time.

   Submit:

   (a) A loglog plot of the decreasing MSE as determined over a 1 s run with a 5 Hz bandlimited signal. Include a plot of $1/N$ fit somewhat to the resulting line.

Notes:

- You need to find the decoders for each new population of neurons.
- You'll get better answers if you run each population more than once and average (perhaps 4 times or so, depending on time).
- Don't worry about adding noise to the spike trains, since the fluctuations due to spiking will introduce some (if you really want, pick random times from a Gaussian with a variance of ~1ms and add that to the spikes). You definitely need to account for the effects of noise with Γ however.
- Be sure to eliminate any constant error due to shift/delay in your MSE calculation (this 'unshifting' can be approximate).
- The PSC can be normalized to 1 in this case, since the ϕ_i take care of magnitude.

5.2 Feedforward transformations

1. Performing linear transformations of scalars using 200 LIF neurons per population. Determine the weight matrices between the necessary populations and decode the spikes from the final population when computing the following transformations.

   Submit:

   (a) Plots of $y = 2x + 1$.

      i. Use a ramp input over the range $[-1, 0]$.

      ii. Use a randomly varying step input over the same range where each step is about .1 s in length.

      iii. Use $x = .2 \sin 6\pi t$.

   (b) Plots of $z = 2y + 0.5x$.

      i. Use $y = .5 \sin 2\pi t$ and $x = \cos 3\pi t$.

      ii. For y use a bandlimited white noise signal at 5 Hz (rms=0.5). For x use a white noise signal at 8 Hz (rms=1).

Note:

- Use the code provided on the course website for computing Γ. Noiseless $\Gamma$is called *Cmatrix*. $\Upsilon$is computed by *moments(:,2)*ones(1,D).*phiTilde.*
- The genActivities function that you need to write returns the neuron activities (firing rates) of all neurons in the population given Rvalues. Rvalues is already projected onto the encoding vectors (i.e. +/- 1 in the 1D case).

- genActivities and its parameters are defined in the comment section of genCmatrix.
- Show a run of 1s for these simulations. Use the same parameters as earlier with $\tau_{psc} = 5ms$.
- Show the expected answer and the estimate of the answer from the decoded spike train.
- Show the input signals as well.
- Overlay spikes of the output population on the graph (just for every 10th neuron).
- In each case note what is interesting (or not) about the results, especially where there are errors. Explain *why* you would expect those results.

2. Performing linear transformations of vectors using LIF neurons. Do exactly what you did for scalars, but with a 2D vector for this transformation:

   (a) Plot $\mathbf{w} = \mathbf{x} - 3\mathbf{y} + 2\mathbf{z} - 2\mathbf{q}$. Do part of the computation in parallel.

      i. Let $\mathbf{x} = [.5, 1]$, $\mathbf{y} = [.1, .3]$, $\mathbf{z} = [.2, .1]$, $\mathbf{q} = [.4, -.2]$.

      ii. Let $\mathbf{x} = [.5, 1]$, $\mathbf{y} = [\sin 4\pi t, .3]$, $\mathbf{z} = [.2, .1]$, $\mathbf{q} = [\sin(4\pi t - .2), -.2]$. Discuss your results especially why and how they stray from the expected answer.

- To use the code for Γ with vectors, you must multiply the provided moments by the encoding vector to get Υ as above.
- When you plot results, be sure to show the vector components over time (i.e. with a time axis).
- Do not plot spikes. Its easiest to put inputs on one graph and outputs on another (overlayed with expected answers for comparison).