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How to build a brain

Dynamics

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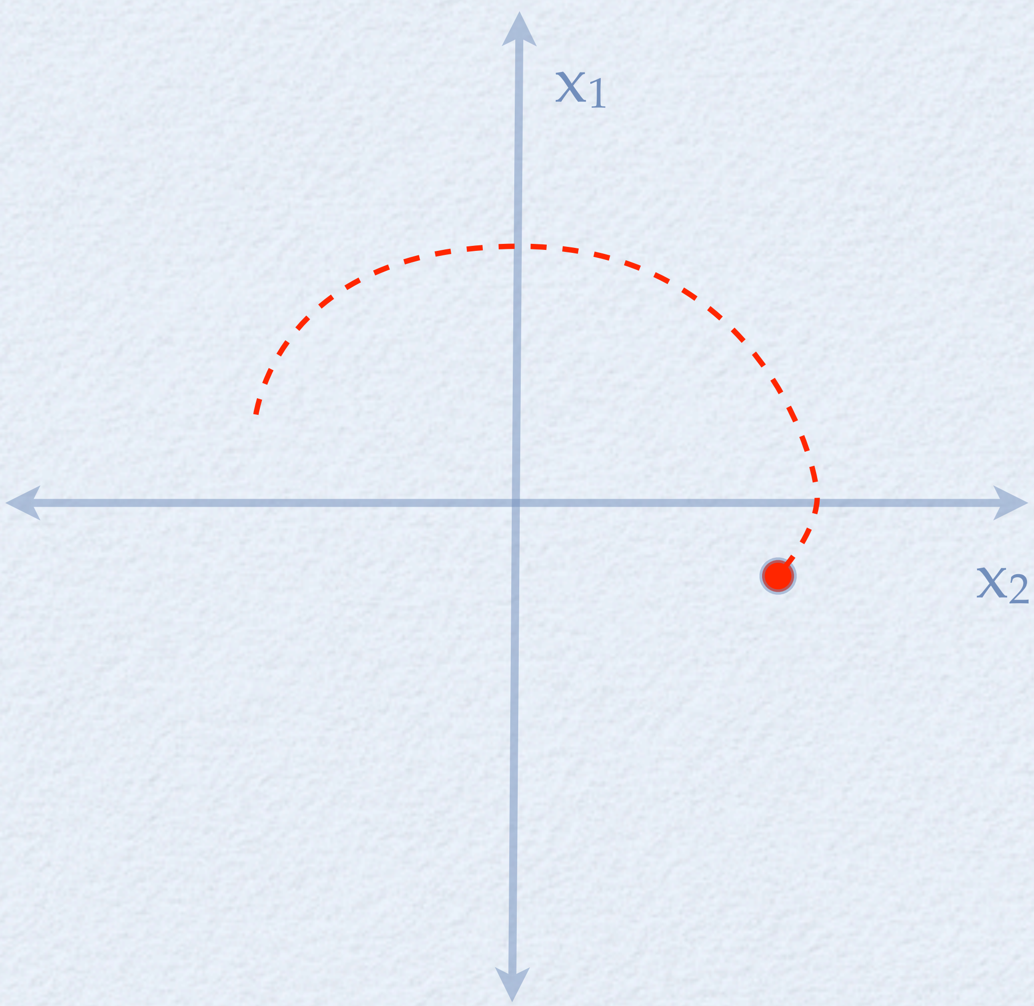
Principle 3: Dynamics

- Adopt control theory (DST) notation
- Add: mapping of a standard representation of dynamics onto neural systems



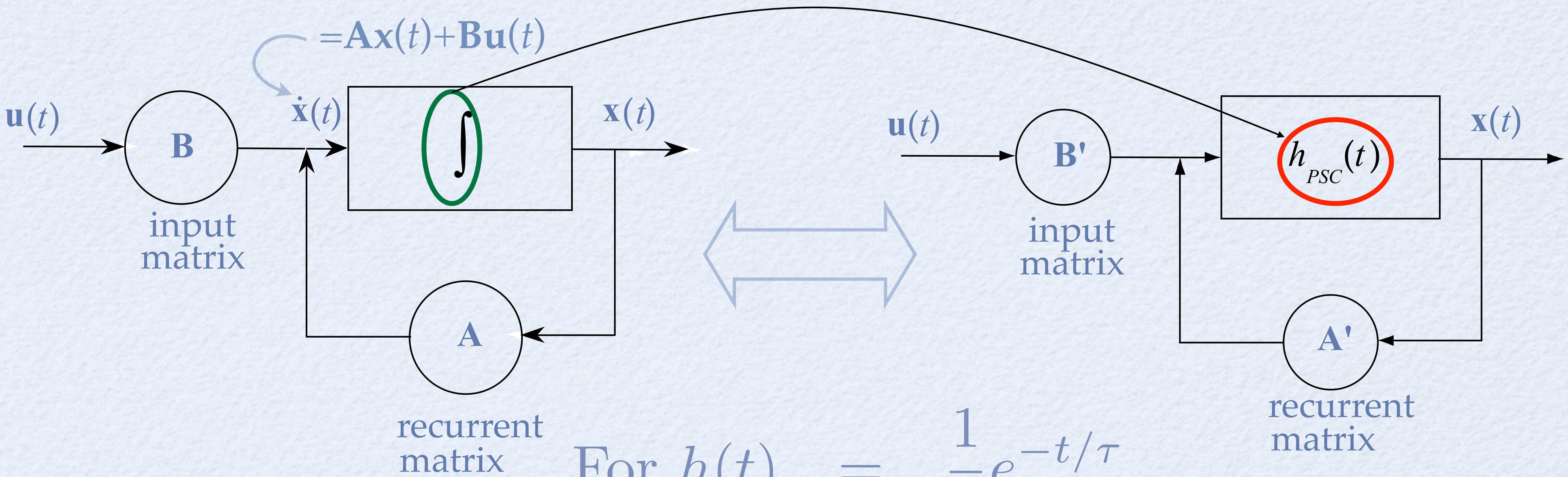
neural dynamics   neural transformation

$$\dot{\mathbf{x}} = f(\mathbf{x}, t)$$

neural representation

Neural Control Theory

- Adapt standard control theory to neurobiological systems

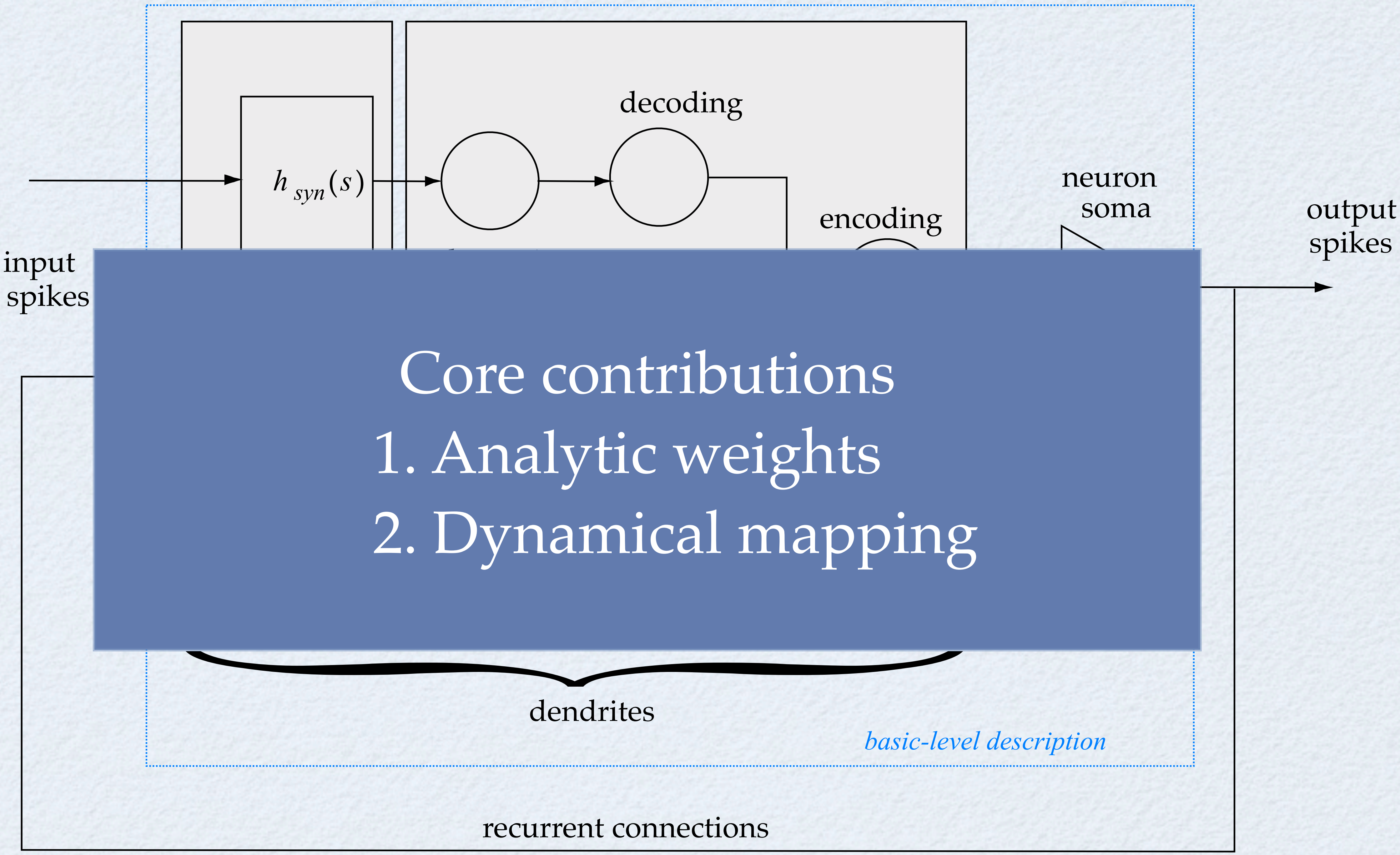


$$\text{For } h(t) = \frac{1}{\tau}e^{-t/\tau}$$

$$\mathbf{A}' = \tau\mathbf{A} + \mathbf{I}$$

$$\mathbf{B}' = \tau\mathbf{B}$$

The NEF defines a *generic* neural subsystem



Core contributions

1. Analytic weights
2. Dynamical mapping

Neural integrator

- NPH & VN turn velocity signals into eye position commands
- Difficult problem to solve, but simple to formulate:
  x is eye position (a scalar)
  u is the eye velocity (a scalar)

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

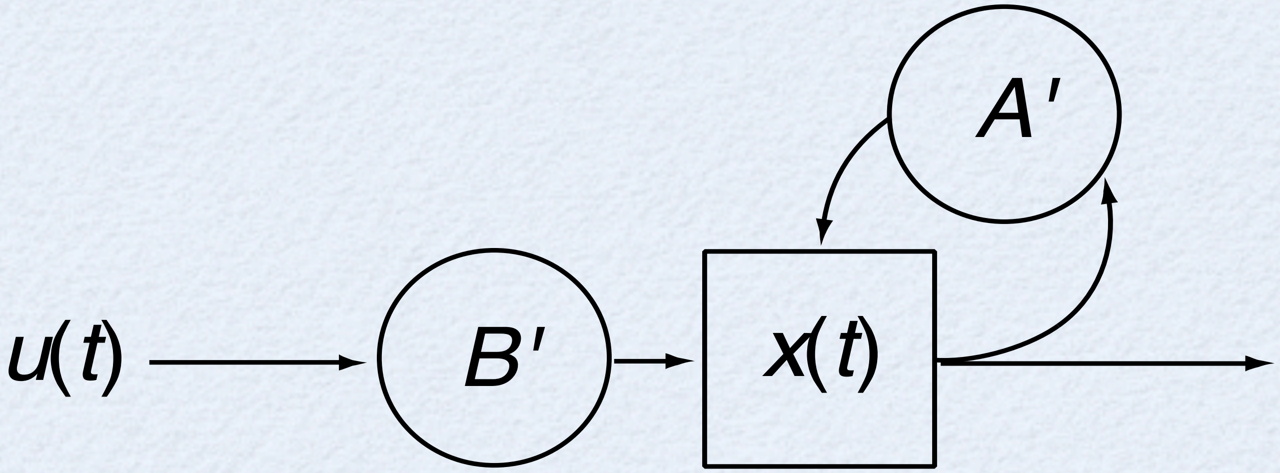
$$\mathbf{A} = 0$$

$$\mathbf{B} = 1$$

Neural integrator

- So, in 'neural control' we have (assuming input and recurrent time constants are equal):

$$\begin{aligned} \text{For } h(t) &= \frac{1}{\tau} e^{-t/\tau} \\ \mathbf{A}' &= \tau \mathbf{A} + \mathbf{I} \\ \mathbf{B}' &= \tau \mathbf{B} \end{aligned}$$



$$\mathbf{A}' = 1$$

$$\mathbf{B}' = \tau$$

Neural integrator

- Substitute this into the encoding equation:

$$a_j(t) = G_j\left[\alpha_j\left\langle x(t)\tilde{\phi}_j\right\rangle + J_j^{bias}\right]$$

- Gives

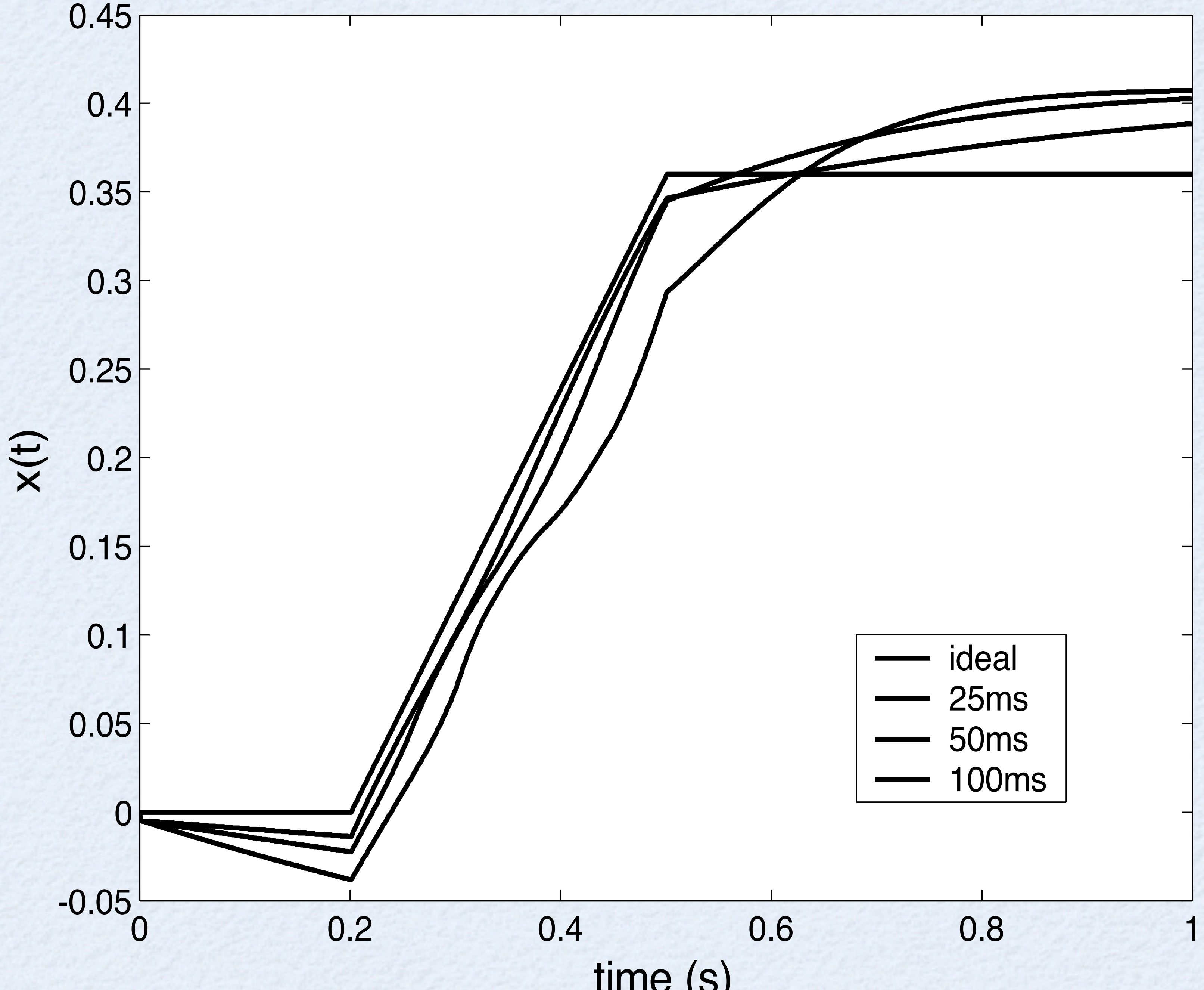
$$\begin{aligned} a_j(t) &= G_j\left[\alpha_j\left\langle h(t) * \tilde{\phi}_j\left[A'\sum_i a_i(t)\phi_i^x + B'u(t)\right]\right\rangle + J_j^{bias}\right] \\ &= G_j\left[h(t) * \left[\sum_i \omega_{ji}a_i(t) + B'\tilde{\phi}_j u(t)\right] + J_j^{bias}\right] \end{aligned}$$

- Where

$$\omega_{ji} = \alpha_j A'\phi_i^x\tilde{\phi}_j$$

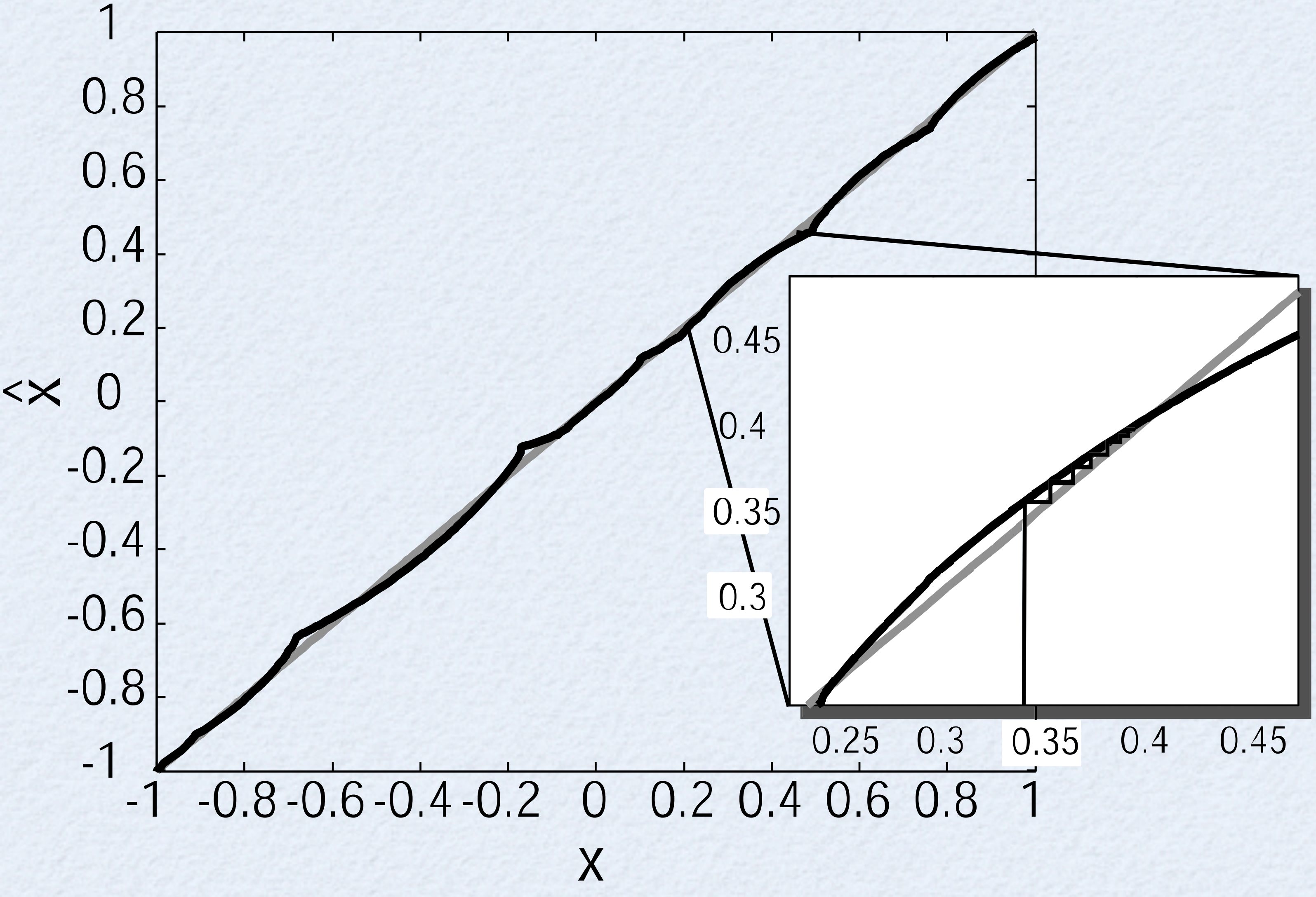
Integrator errors

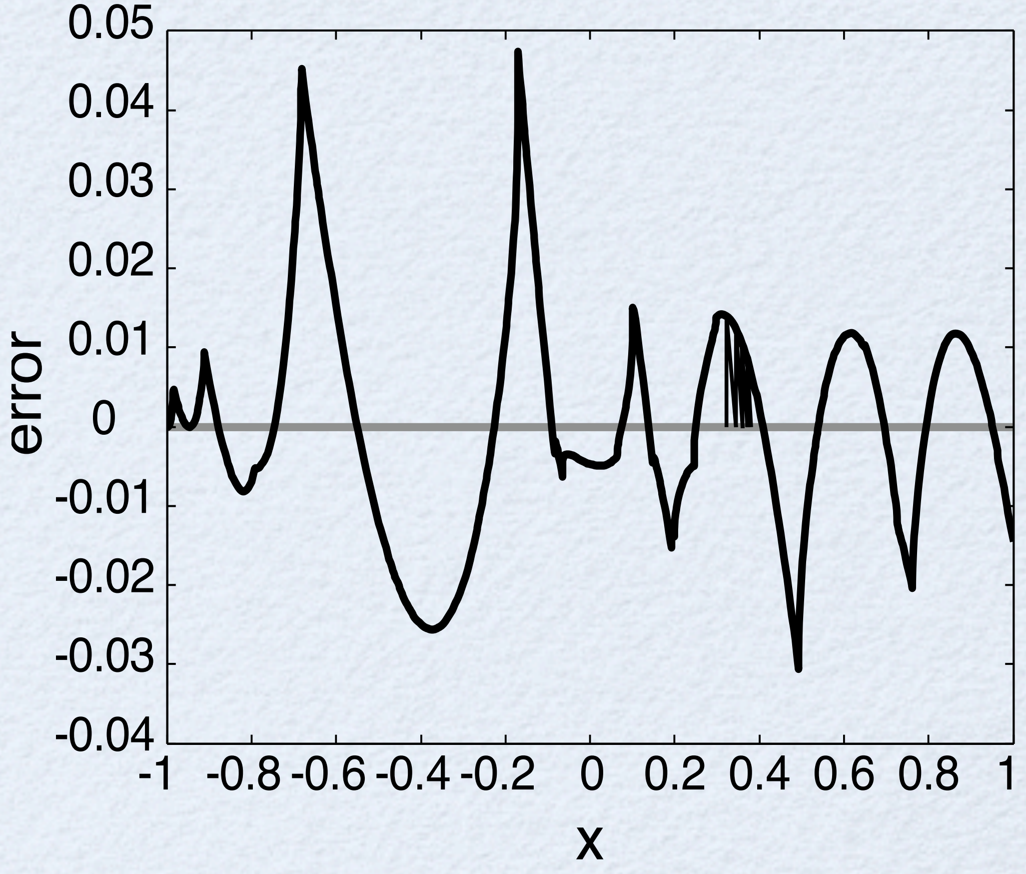
- Synaptic time constant



Integrator errors

- Representational error

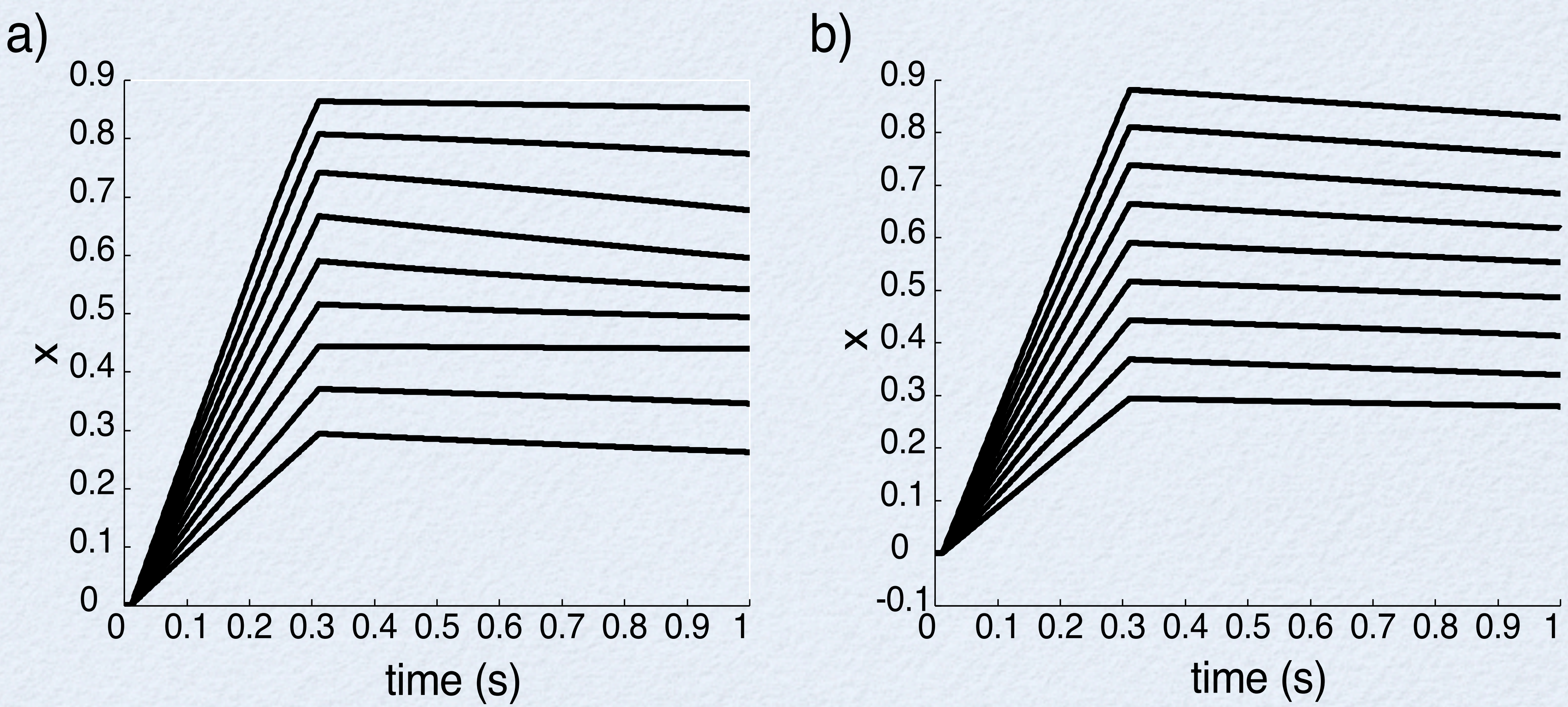




Humans and goldfish

- Humans (left) have more neurons, and hence slower drift (70s vs 10s)

- Both have centripital drift (i.e. $A'<1$)



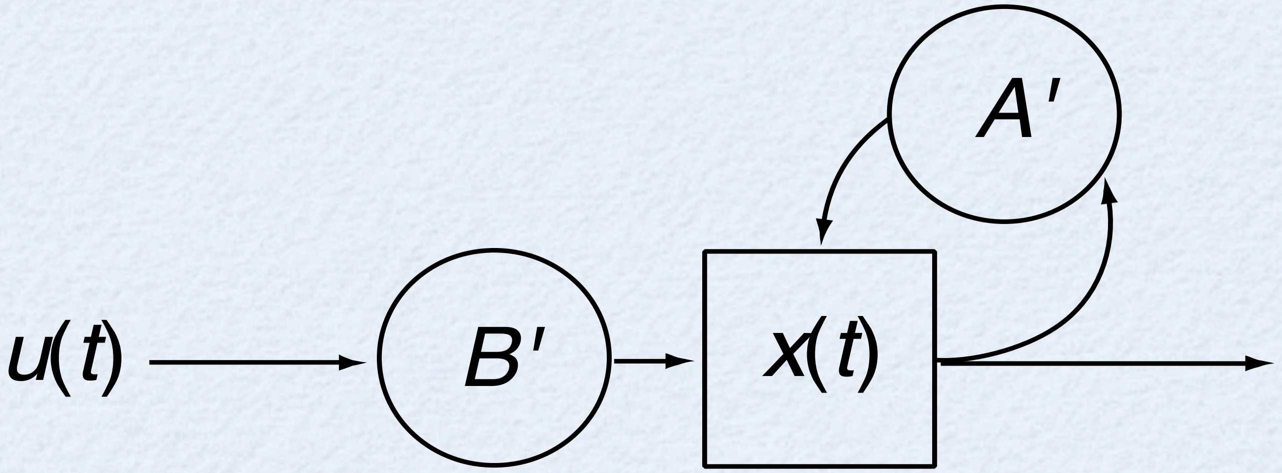
Integrators*

- Are ubiquitous in brains because they:
  - Store information over time (working memory)
  - Clean up noisy input
  - Perform MAP inference
  - Extend the past into the future
  - Are an example of the even more ubiquitous attractor network

Controlled integrator

- Let A′ be a runtime variable

$$\mathbf{A}' = \mathbf{A}'(t)$$
$$\mathbf{B}' = \tau$$



Tunable filter

- Tune A′ directly with a neural ensemble

$$A'(t) = \sum_l b_l(t)\phi_l^{A'}$$

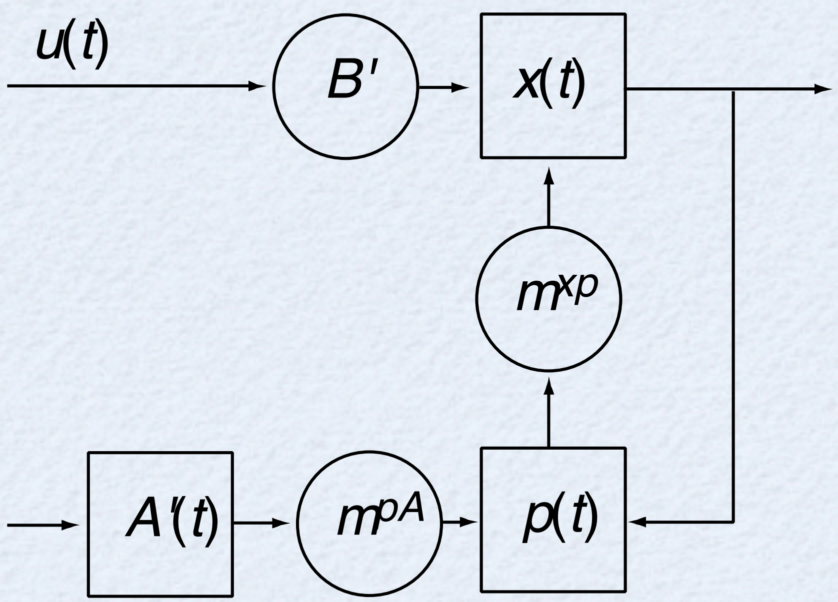
- Direct substitution gives

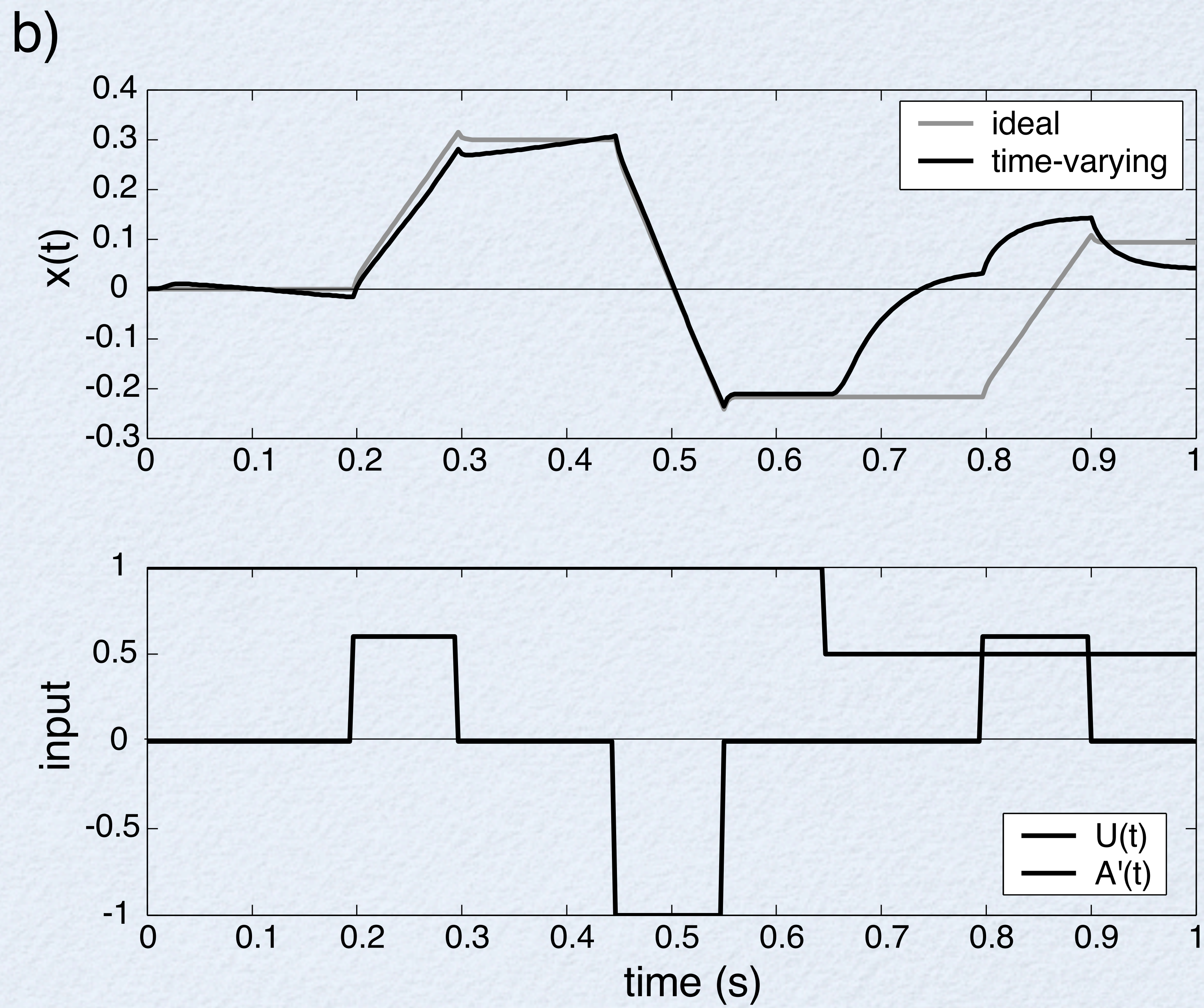
$$a_j(t) \quad = \quad G_j\left[\alpha_j\left(h(t) * \tilde{\phi}_j\left[\sum_l b_l(t)\phi_l^{A'}\sum_i a_i(t)\phi_i^x + B'u(t)\right]\right) + J_j^{bias}\right]$$

- Using an intermediate population gives

$$a_j(t) = G_j\left[\alpha_j\left(h(t) * \tilde{\phi}_j\left[\sum_m c_m(t)\boldsymbol{\phi}_{\boldsymbol{m}}^{\mathbf{p}} + B'u(t)\right]\right) + J_j^{bias}\right]$$

Tunable filter





Tunable filter*

- Using a single 2D population is most efficient