How to build a brain

Linear Transformations

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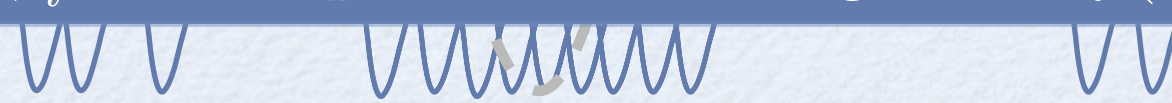
Principle 2: Transformation

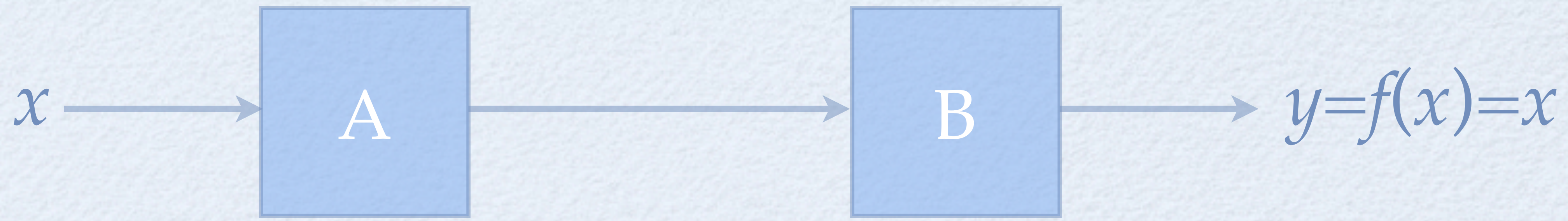
encoding $\sum_n \delta_i(t - t_n) = G_i(\mathbf{e}_i \cdot \mathbf{x}(t))$

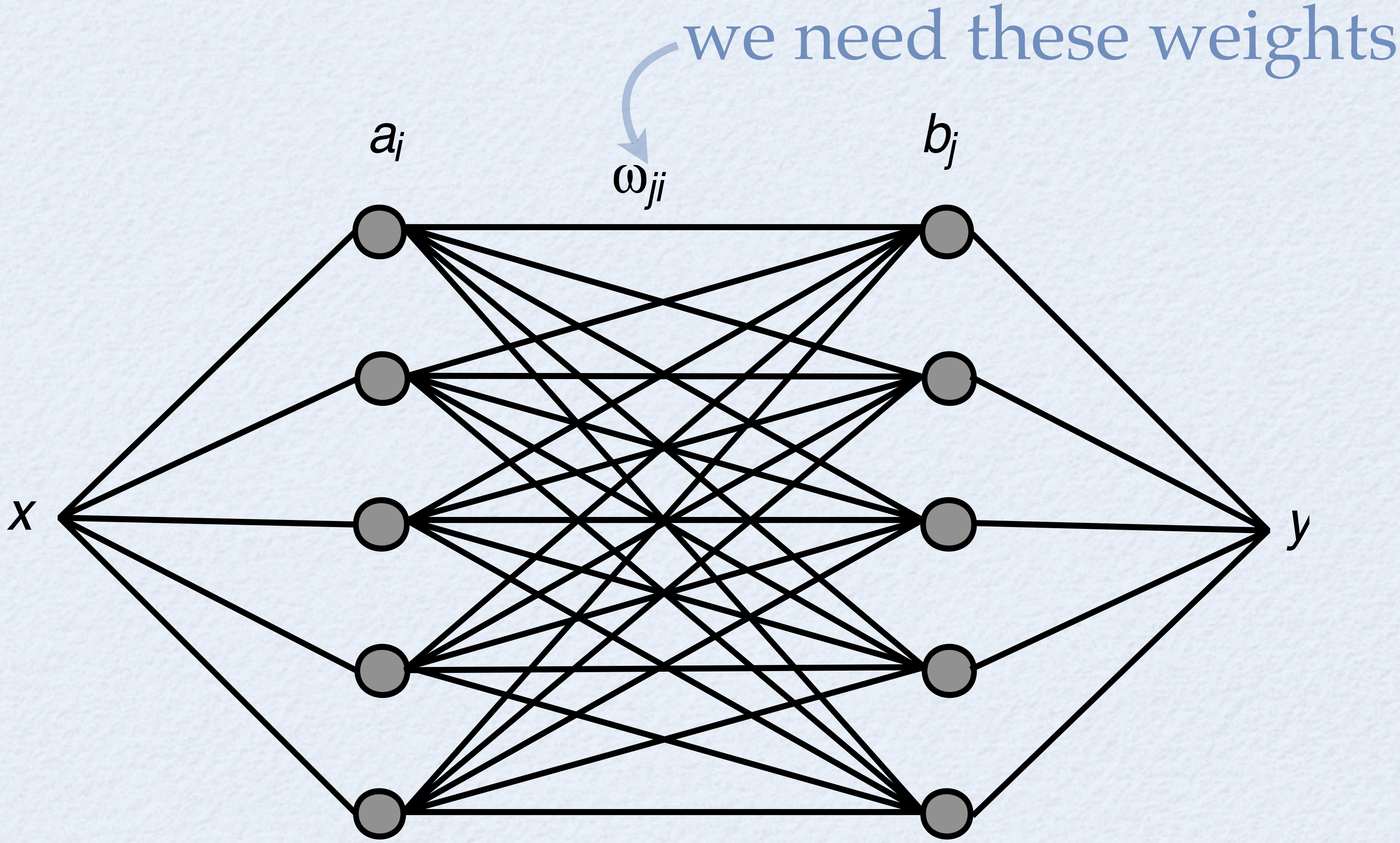
decoding $\hat{f}(\mathbf{x}) = \sum_{i,n} h_i(t - t_n)\phi_i^{f(\mathbf{x})}$

$\delta_i(t - t_n)$	spikes from neuron i at time t_n
G_i	neuron model generating spikes
$\mathbf{e}_i$	preferred direction vector
$\mathbf{x}(t)$	stimulus signal
$\hat{f}(\mathbf{x})$	computed function of stimulus
$h_i(t - t_n)$	PSCs convolved with spikes
$\phi_i^{f(\mathbf{x})}$	optimal linear weights for $f(\mathbf{x})$



A communication channel





Connection weights

- Define the representations of both pops:

$$
\begin{aligned}
a_i(x) &= G_i\left[J_i(x)\right] & b_j(y) &= G_j\left[J_j(y)\right] \\
&= G_i\left[\alpha_i \tilde{\phi}_i x + J_i^{bias}\right] & &= G_j\left[\alpha_j \tilde{\phi}_j y + J_j^{bias}\right] \\
\hat{x} &= \sum_i a_i(x)\phi_i^x & \hat{y} &= \sum_j b_j(y)\phi_j^y
\end{aligned}
$$

- Define the computation: $y=f(x)=x$

- Substitute our estimate of x into b repn

NB: $\mathbf{e}$ and $\tilde{\phi}$ are the same!

Connection weights

- Substituting: $y = x \approx \hat{x}$

$$
\begin{aligned}
b_j(x) &= G_j\left[\alpha_j \tilde{\phi}_j x + J_j^{bias}\right] \\
&= G_j\left[\alpha_j \tilde{\phi}_j \sum_i a_i(x)\phi_i^x + J_j^{bias}\right] \\
&= G_j\left[\sum_i \omega_{ji} a_i(x) + J_j^{bias}\right]
\end{aligned}
$$

$$\omega_{ji} = \alpha_j \tilde{\phi}_j \phi_i^x$$

With spikes*

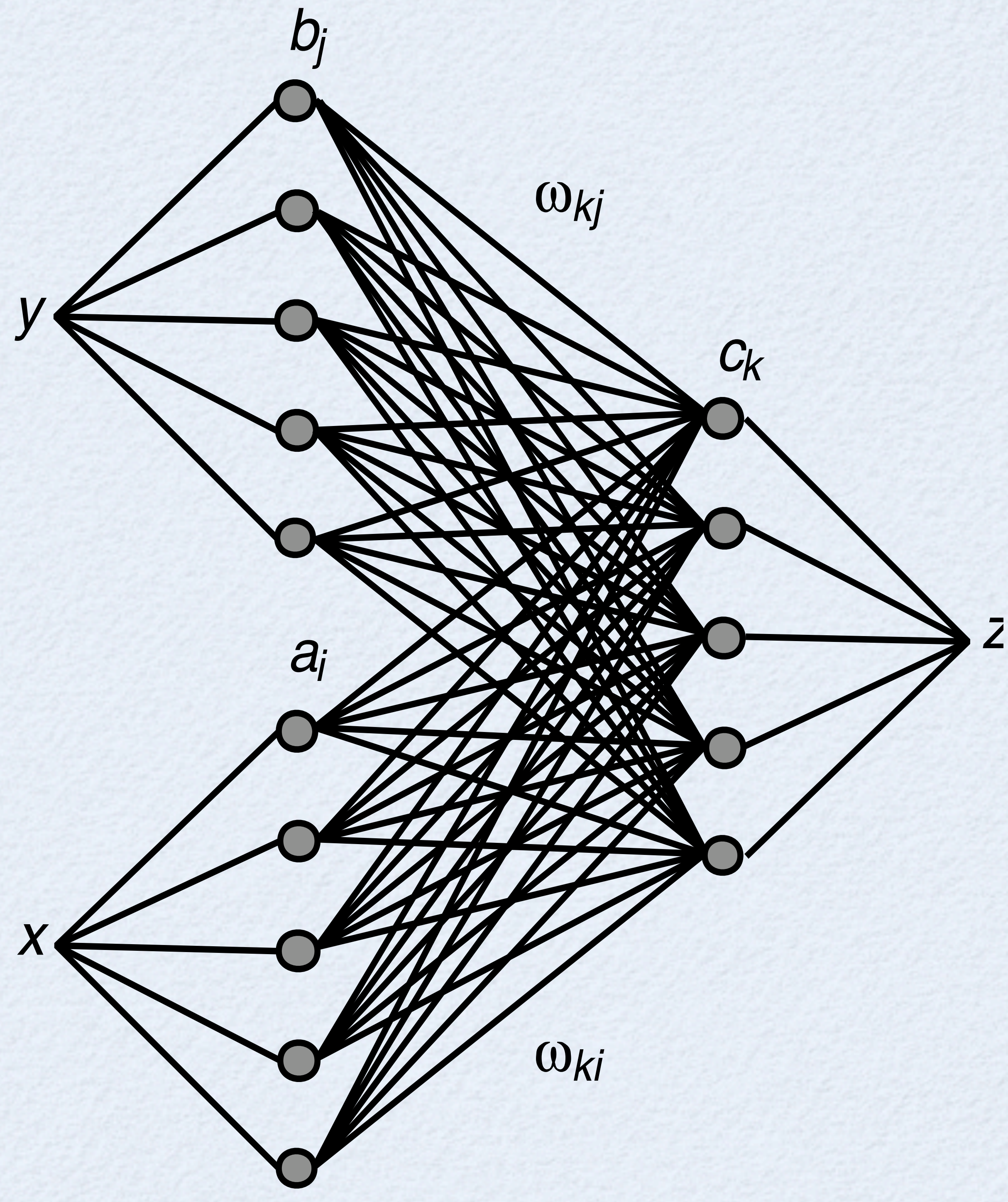
- Write the spiking estimate

$$\begin{aligned}\hat{x}(t) &= \sum_i a_i(x(t))\phi_i^x \\ &= \sum_{i,n} h_i(t - t_{in})\phi_i^x\end{aligned}$$

- Then do the same substitution:

$$\begin{aligned}b_j(x(t)) &= G_j\left[\alpha_j\tilde{\phi}_j x(t) + J_j^{bias}\right] \\ &= G_j\left[\alpha_j\tilde{\phi}_j \sum_{i,n} h_i(t - t_{in})\phi_i^x + J_j^{bias}\right] \\ &= G_j\left[\sum_{i,n} \omega_{ji} h_i(t - t_{in}) + J_j^{bias}\right]\end{aligned}$$

Adding scalars



Doing the math

- Volunteer?

$$
\begin{aligned}
c_k(x+y) &= G_k\left[\alpha_k \tilde{\phi}_k (x+y) + J_k^{bias}\right] \\
&= G_k\left[\alpha_k \tilde{\phi}_k \left(\sum_i a_i(x)\phi_i^x + \sum_j b_j(y)\phi_j^y\right) + J_k^{bias}\right] \\
&= G_k\left[\sum_i \omega_{ki} a_i(x) + \sum_j \omega_{kj} b_j(y) + J_k^{bias}\right]
\end{aligned}
$$

$$
\omega_{ki} = \alpha_k \tilde{\phi}_k \phi_i^x \quad \omega_{kj} = \alpha_k \tilde{\phi}_k \phi_j^y
$$

Recipe for linear trans.*

- 1. Define the repn (enc/dec) for all variables involved in the operation.
- 2. Write the transformation in terms of these variables.
- 3. Write the transformation using the decoding expressions for all variables except the output variable.
- 4. Substitute this expression into the encoding expression of the output variable.

Vectors

- Nothing new. Representation:

$$a_i(\mathbf{x}) = G_i\left[\alpha_i \left\langle \tilde{\phi}_i \mathbf{x} \right\rangle_m + J_i^{bias}\right]$$
$$\hat{\mathbf{x}} = \sum_i a_i(\mathbf{x})\phi_i^{\mathbf{x}}$$

- Transformation

$$\mathbf{z} = C_1\mathbf{x} + C_2\mathbf{y}$$

Vectors

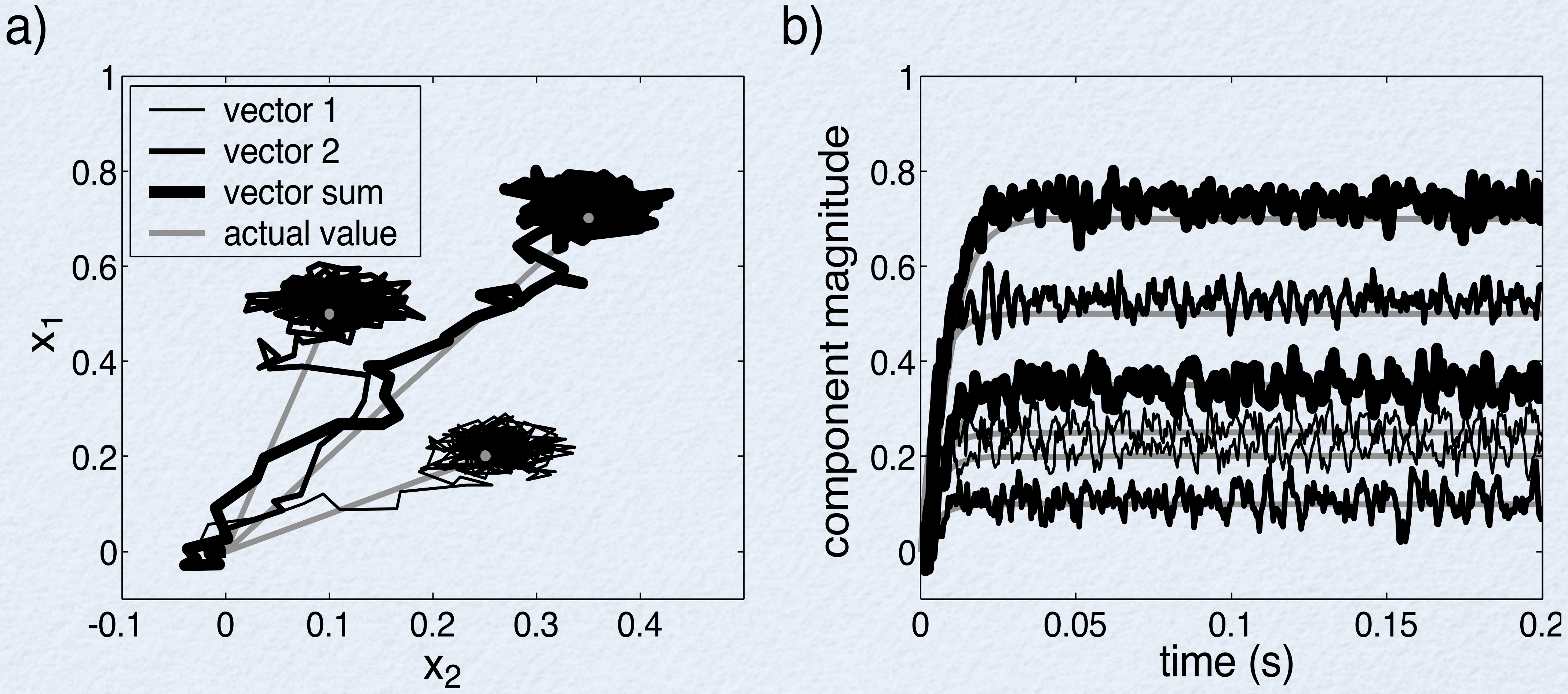
- Substitution

$$
\begin{aligned}
c_k(C_1\mathbf{x}+C_2\mathbf{y}) &= G_k\left[\alpha_k\left\langle\tilde{\phi}_k(C_1\mathbf{x}+C_2\mathbf{y})\right\rangle_m + J_k^{bias}\right] \\
&= G_k\left[\alpha_k\left\langle\tilde{\phi}_k\left(C_1\sum_i a_i(\mathbf{x})\phi_i^{\mathbf{x}} + C_2\sum_j b_j(\mathbf{y})\phi_j^{\mathbf{y}}\right)\right\rangle_m + J_k^{bias}\right] \\
&= G_k\left[\sum_i \omega_{ki}a_i(\mathbf{x}) + \sum_j \omega_{kj}b_j(\mathbf{y}) + J_k^{bias}\right]
\end{aligned}
$$

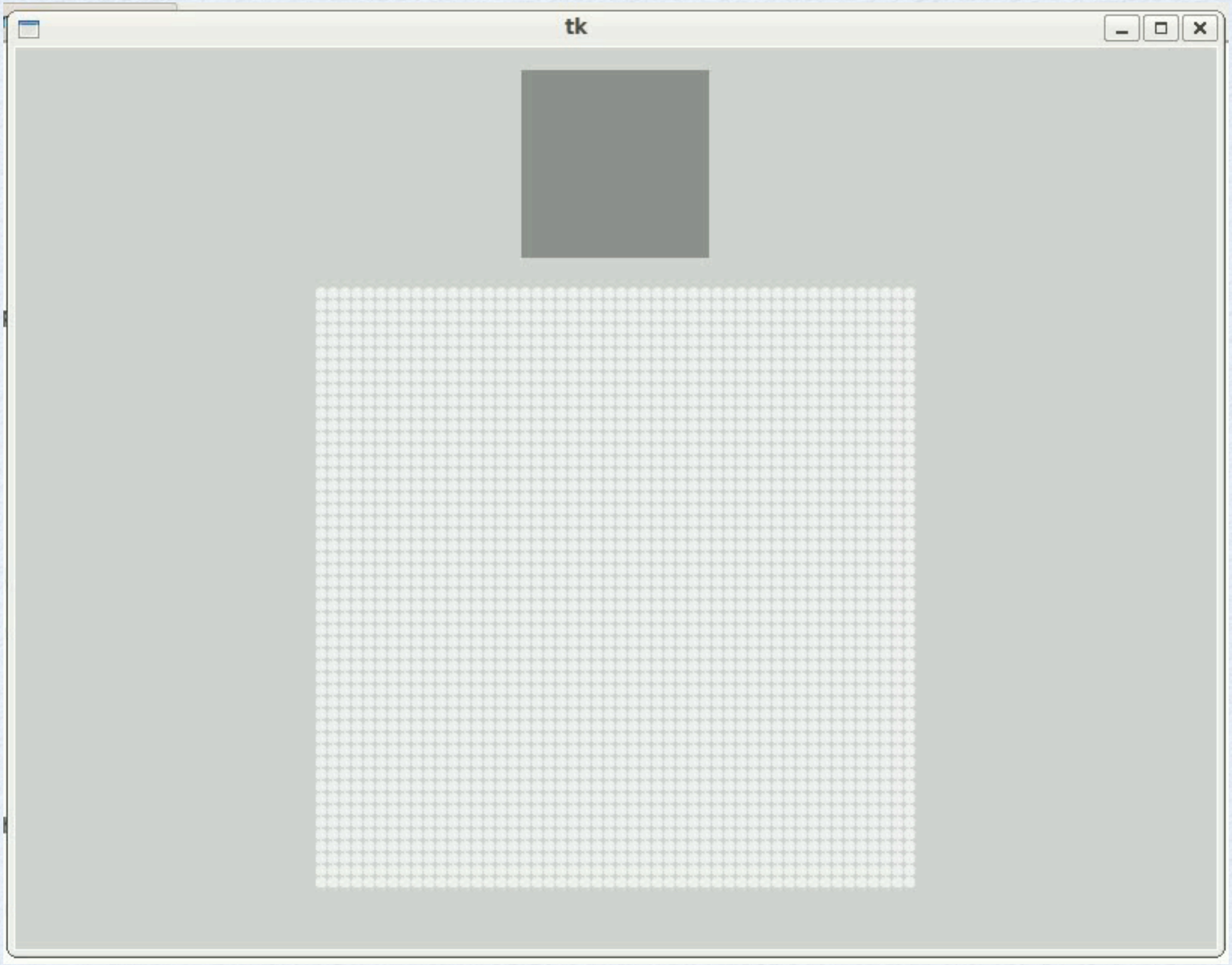
$$
\omega_{ki} = \alpha_k C_1\left\langle\tilde{\phi}_k\phi_i^{\mathbf{x}}\right\rangle_m \quad \omega_{kj} = \alpha_k C_2\left\langle\tilde{\phi}_k\phi_j^{\mathbf{y}}\right\rangle_m
$$

Vector addition

- a) vector space; b) components



25D Vector Representation



Representational hierarchy*

Kind	*Encoder* $(a_i(x))$	*Decoder* $(\hat{x})$
Scalar (1)	$G_i\left[\alpha_i x + J_i^{bias}\right]$	$\sum_i a_i(x)\phi_i$
Vector (N)	$G_i\left[\alpha_i \left\langle \tilde{\phi}_i[n]x[n] \right\rangle_n + J_i^{bias}\right]$	$\sum_i a_i(x[n])\phi_i[n]$
Function (∞)	$G_i\left[\alpha_i \left\langle \tilde{\phi}_i(\nu)x(\nu) \right\rangle_\nu + J_i^{bias}\right]$	$\sum_i a_i(x(\nu))\phi_i(\nu)$
Vector Field ($\infty \times N$)	$G_i\left[\alpha_i \left\langle \tilde{\phi}_i(\nu,[n])x(\nu,[n]) \right\rangle + J_i^{bias}\right]$	$\sum_i a_i(x(\nu,[n])\phi_i(\nu,[n])$