A Biologically Plausible Model of Mental Multiplication

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Introduction

Mental multiplication is an advanced, abstract cognitive task that separates adults from non-human animals, AI systems, and young children. We present a biologically and psychologically plausible spiking neural model of simple mental multiplication, expanding on previous work on mental addition [1, 2].

Background

People primarily use three main solution strategies [3–9]:

- **Retrieval**: fact recall from memory (first attempt).
- **Repeated addition (counting)**: iteratively adding one factor the number of times of the other (first ‘backup’).
- **Rules**: shortcuts for 0- and 1-multiplication (special cases).

Strategy choice depends on the problem. Only when retrieval fails are more involved ‘backup’ strategies invoked.

People consistently exhibit two main patterns [3–9]:

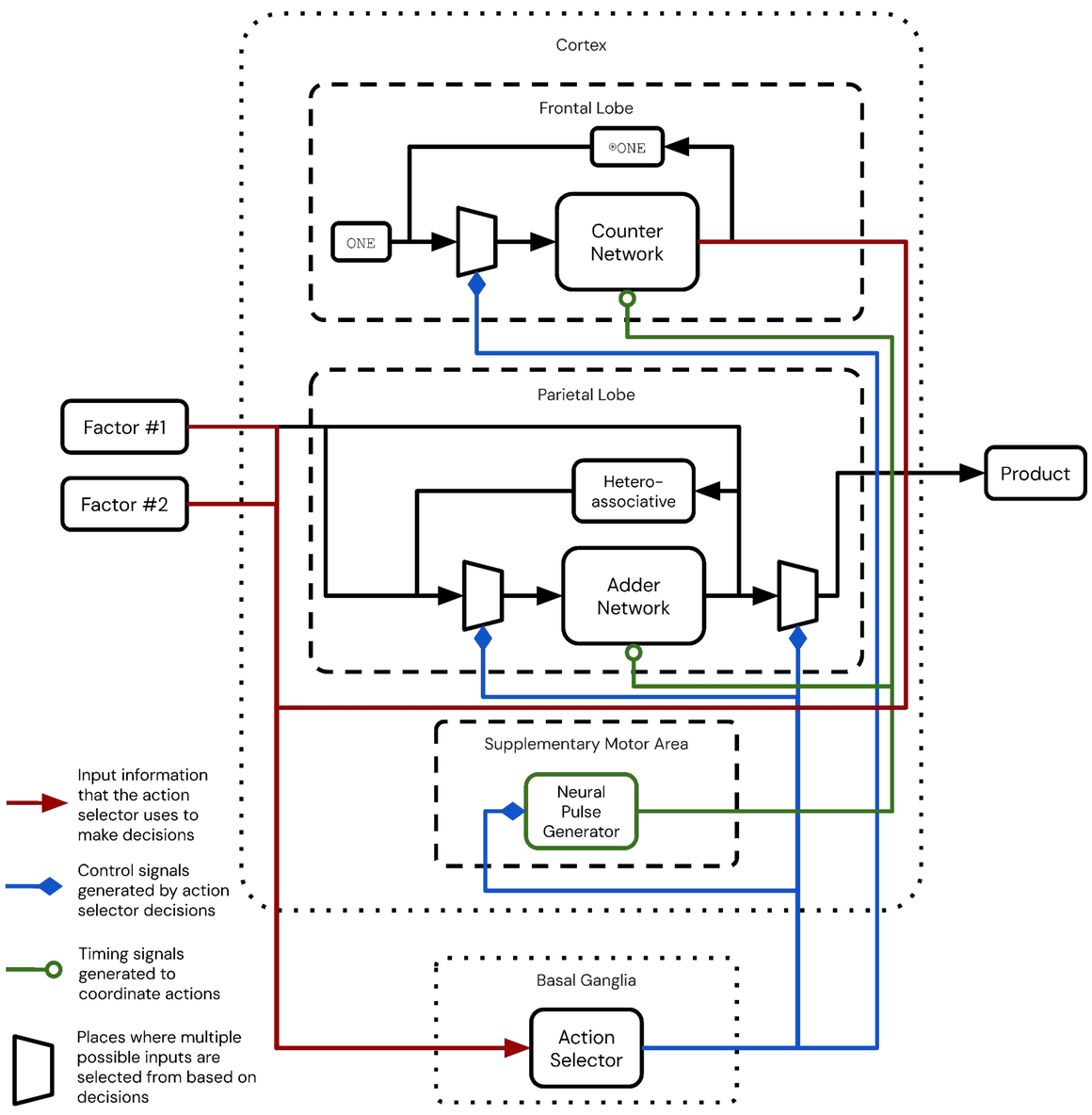
- **Problem-size effect**: ‘smaller’ problems are easier/faster.
- **Outliers**: 0-, 1-, 5-multiplication and ties are easier/faster.

We propose that people use the following algorithm:

- If the person has seen the problem before and is confident enough in a memorized solution, they use retrieval. Otherwise, they manually compute the solution using a backup strategy. If a rule can be applied, they apply that rule; otherwise, they perform repeated addition.

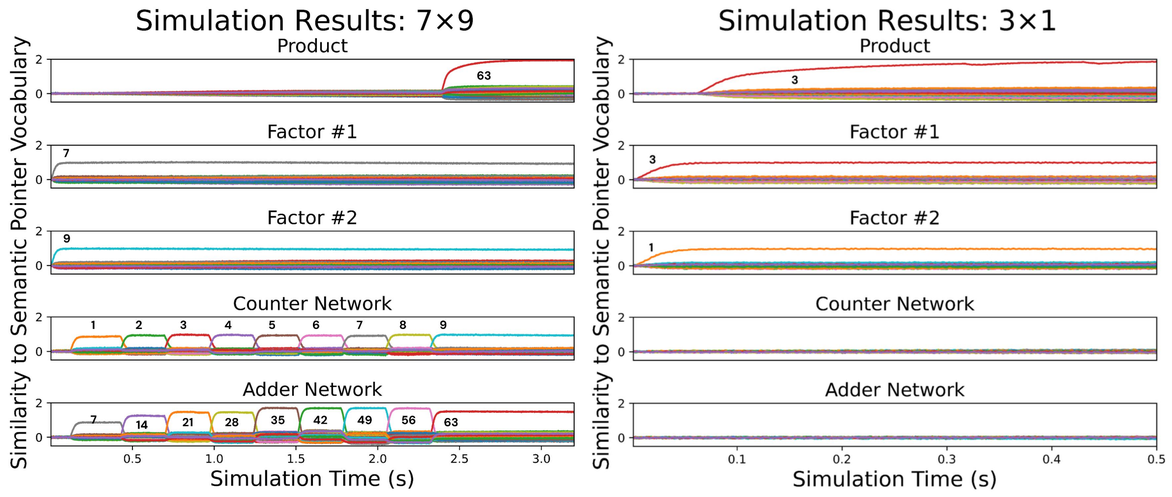
Model

The model leverages the Semantic Pointer Architecture (SPA) [10] and Neural Engineering Framework (NEF) [11] and is implemented in 200K–2M spiking neurons using Nengo [12].

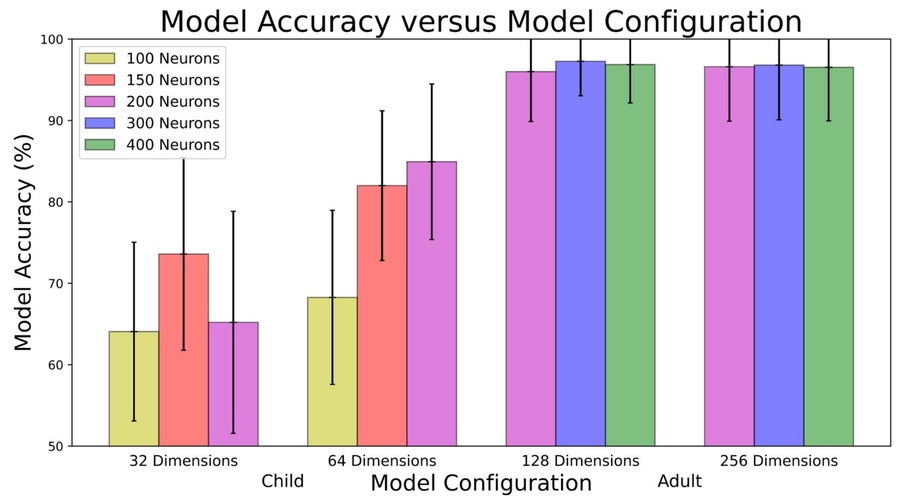


Results

Qualitatively, the model was capable of correctly executing the repeated addition and rules strategies.



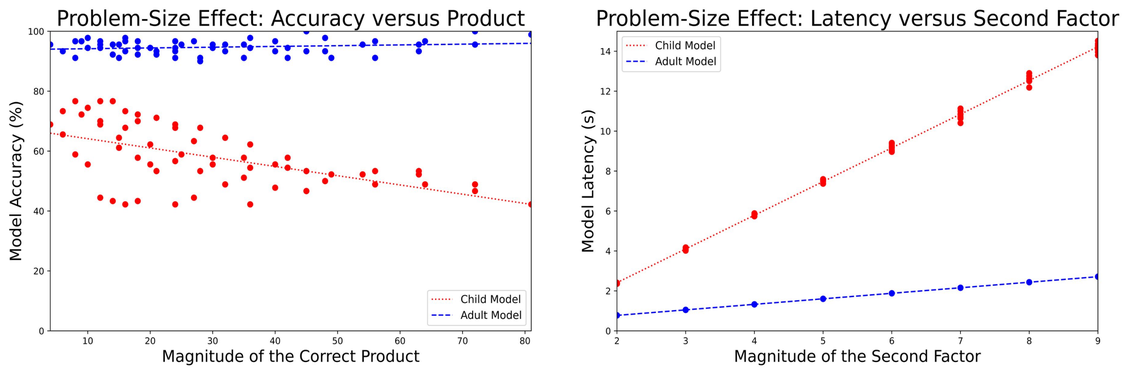
Quantitatively, model performance scaled with resources.



The performance of separate ‘child’ and ‘adult’ versions of the model matched data obtained from studies on people.

Study	Subjects	Rules		Repeated Addition	
		Accuracy (%)	Latency (s)	Accuracy (%)	Latency (s)
Lemaire and Siegler (1995)	French Children	—	—	71	11.8
Siegler (1988)	American Children	—	—	59	23.3
Ours	Child Model	99.9	0.322	57.9	6.795
LeFevre et al. (1996)	Canadian Adults	98.9	0.895	97.2	1.490
Ours	Adult Model	100.0	0.324	94.7	1.747

The model reproduced the problem-size effect and outliers.



Conclusion

We presented a novel, biologically and psychologically plausible, spiking neural model of mental multiplication. The model:

- Implemented key strategies used by people.
- Replicated performance levels and trends found in people.
- Had tunable accuracy that could be made perfect.

References

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