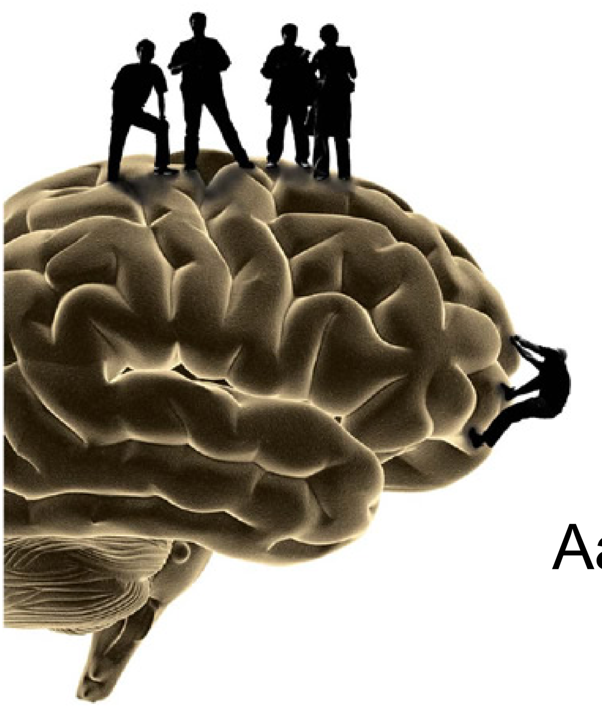

Extending the NEF for Nonideal Silicon Synapses

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Motivation

The Neural Engineering Framework (NEF) is a theory used to map high-level computations onto neuromorphic hardware. It assumes an ideal first-order lowpass synapse (spike filter).

$$H(s) = \frac{1}{\tau s + 1}$$

Silicon (and biological) synapses exhibit higher-order dynamics and heterogeneity.

$$H_j(s) = \frac{\gamma_j \left(1 - e^{-\epsilon_j s}\right) s^{-1}}{\left(\tau_{j,1} s + 1\right)\left(\tau_{j,2} s + 1\right)}$$

We account for these properties to improve the accuracy of our neuromorphic systems.

Neural Engineering Framework (NEF)

Principle 1 – A vector $\mathbf{x}(t)$ is represented by the spiking activity of a population of neurons.

$$\delta_i(t) = G_i \left[\alpha_i \mathbf{e}_i \cdot \mathbf{x}(t) + \beta_i\right]$$

Principle 2 – We optimize for decoding vectors that transform this activity into a function of $\mathbf{x}(t)$.

$$\sum_{i=1}^{n} (\delta_i * h)(t)\mathbf{d}_i^{\mathbf{f}(\mathbf{x})} \approx (\mathbf{f}(\mathbf{x}) * h)(t)$$

Principle 3 – And then harness the dynamics of the synapse to implement nonlinear dynamical systems.

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{u}, \quad \mathbf{y} = \mathbf{g}(\mathbf{x})$$

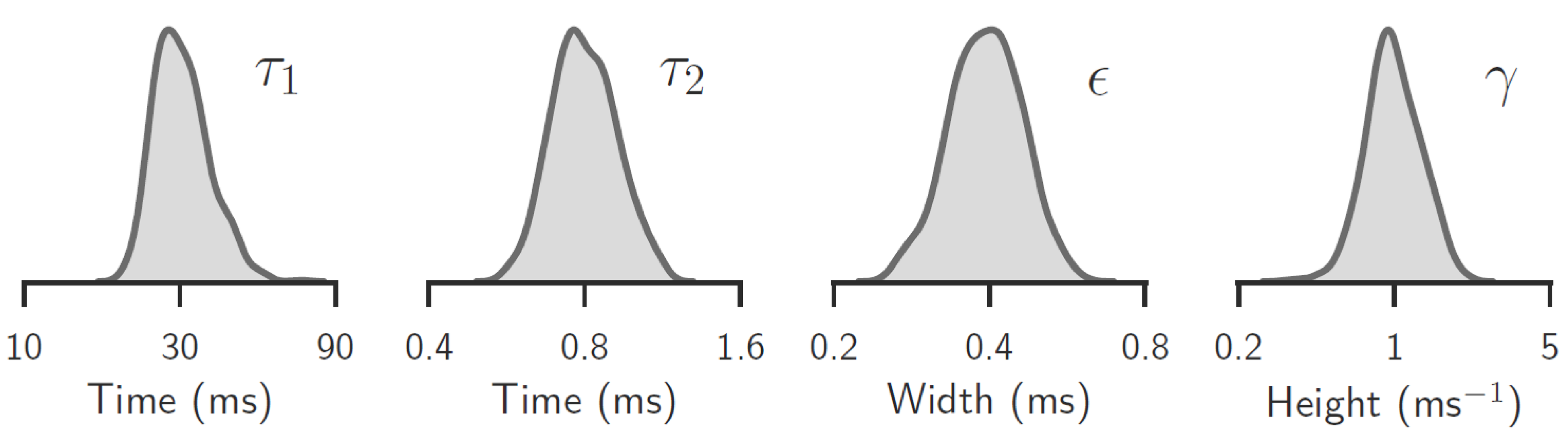
Full Mapping / Extension

$$\mathbf{w}_j = \mathbf{\Phi}\mathbf{\Gamma}_j,$$

$$\mathbf{\Phi} := [\mathbf{x}\ \dot{\mathbf{x}}\ \ddot{\mathbf{x}}],$$

$$\mathbf{\Gamma}_j := (\epsilon_j \gamma_j)^{-1} \begin{bmatrix} 1 \\ \tau_{j,1} + \tau_{j,2} + \epsilon_j/2 \\ \tau_{j,1}\tau_{j,2} + (\epsilon_j/2)(\tau_{j,1} + \tau_{j,2}) \end{bmatrix}$$

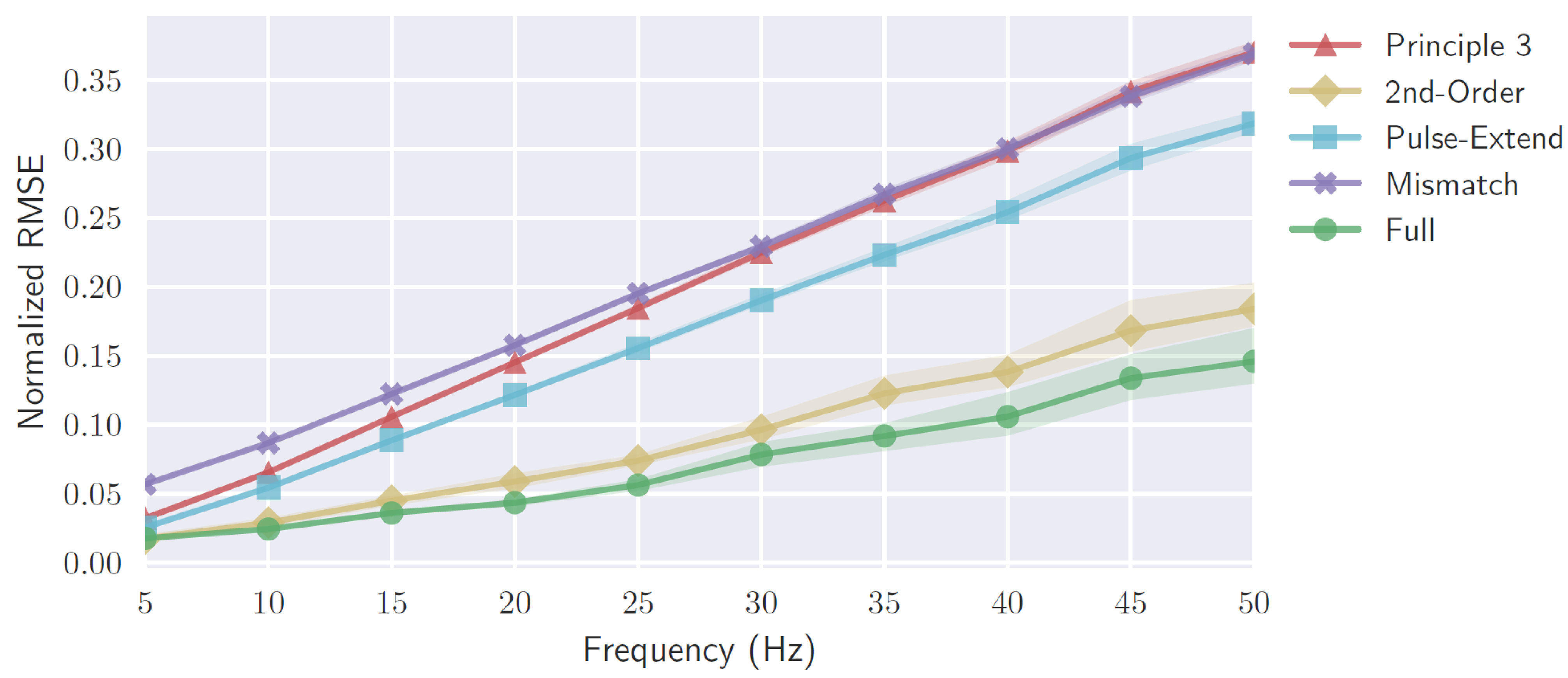
Synapse Parameter Distribution



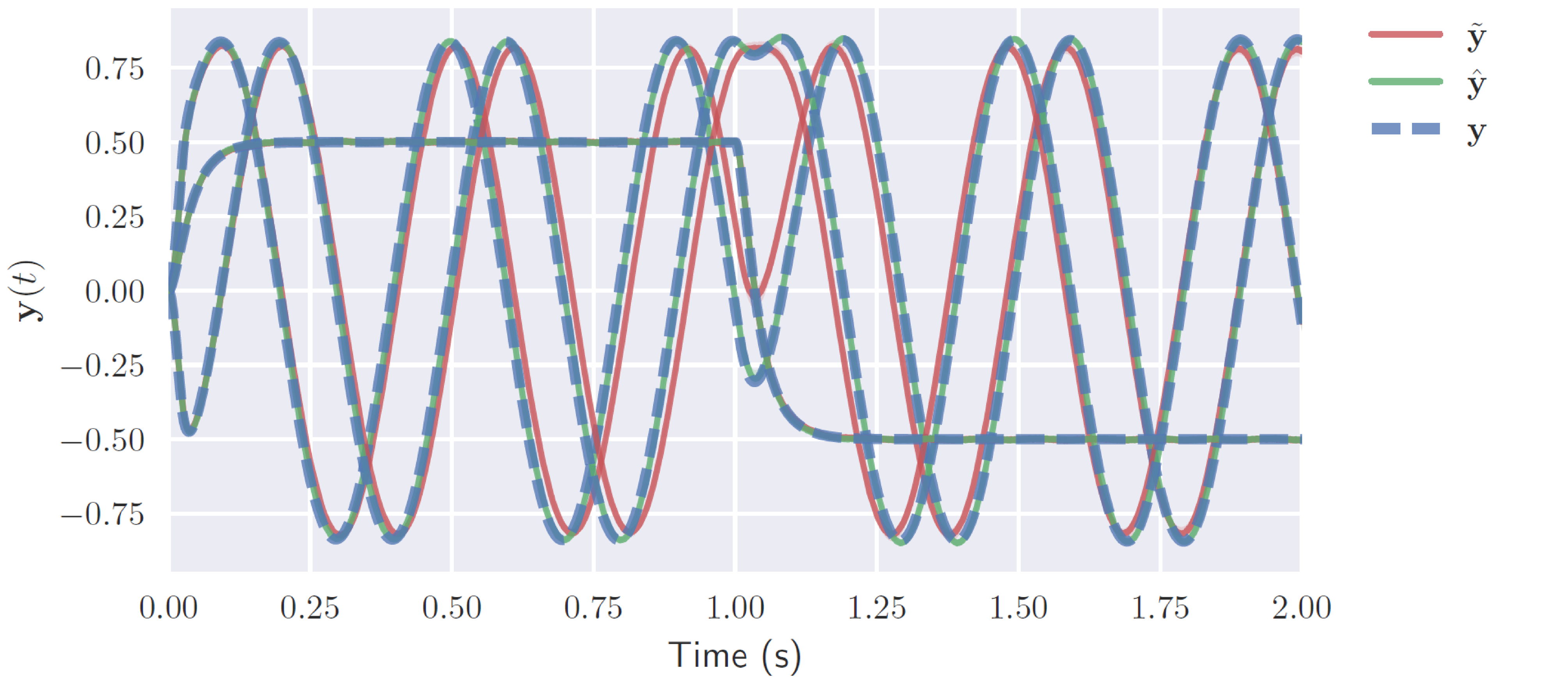
Conclusions

Our theory enables a more accurate mapping of nonlinear dynamical systems onto a recurrently connected neuromorphic architecture. This paves the way toward understanding how physical primitives may be exploited to support useful computations in neuromorphic hardware.

Results

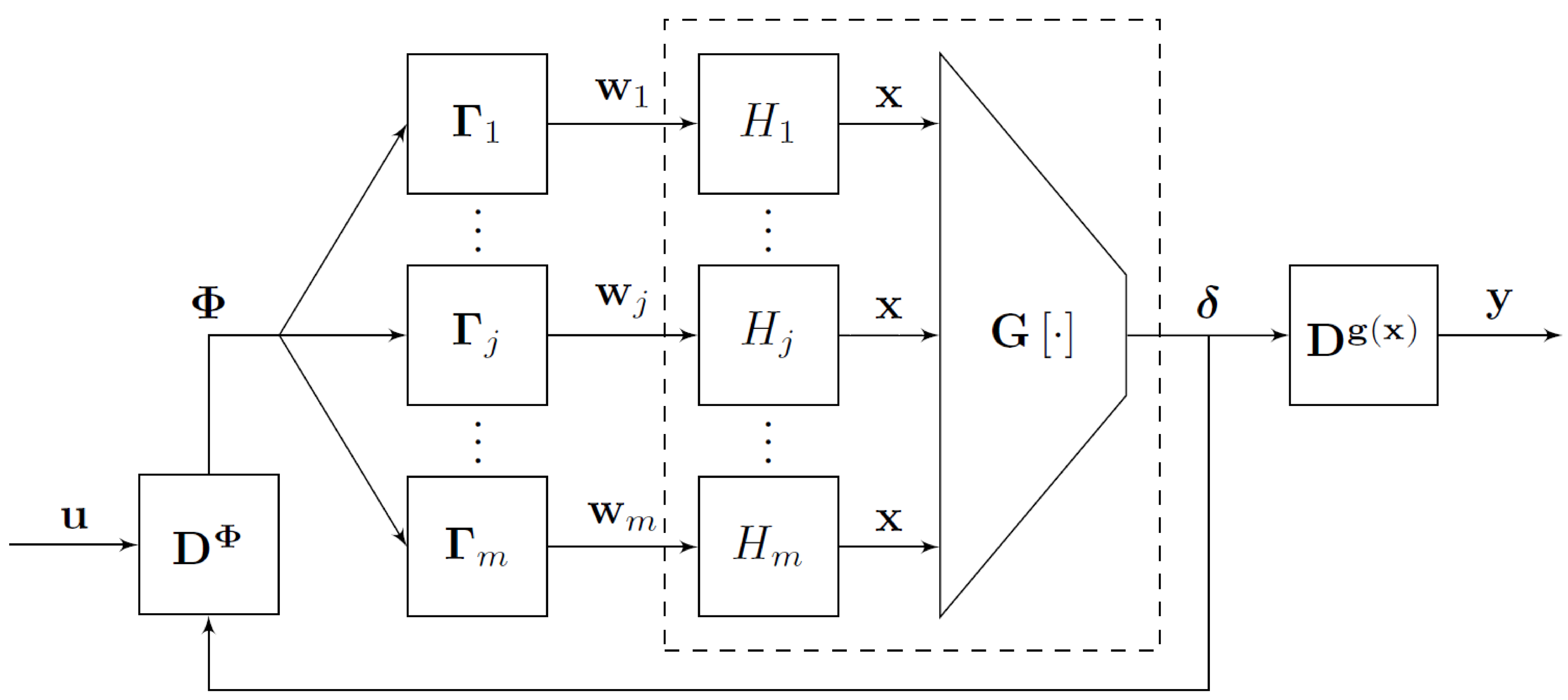


Effect of each NEF extension, applied to a simulated integrator. Our full extension provides an overall 63% reduction in error. The largest improvement comes from accounting for the second-order dynamics.



Output of a controlled oscillator. Our extension ($\hat{\mathbf{y}}$) improves the accuracy of Principle 3 ($\tilde{\mathbf{y}}$) by 73% compared to the ideal ($\mathbf{y}$).

Model Architecture



Synapse Circuit