Real Time Vehicle Tracking
Using Background Subtraction and Kalman Filters

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May 2009
Overview

- **Introduction**
  - Object Tracking
  - Vehicle Tracking

- **Theory & Implementation**
  - Segmentation
  - Tracking

- **Results**

- **Q & A**
Introduction – Object Tracking

- Object Tracking
  - Object representation
  - Feature Selection
  - Object Detection
  - Tracking
Introduction – Object Tracking

- Object Representation

Fig. 1. Object representations. (a) Centroid, (b) multiple points, (c) rectangular patch, (d) elliptical patch, (e) part-based multiple patches, (f) object skeleton, (g) control points on object contour, (h) complete object contour, (i) object silhouette.
Introduction – Object Tracking

- Feature Selection for Tracking
  - Colour
  - Edges
  - Optical Flow
  - Texture
Introduction – Object Tracking

- **Object Detection**

| Point detectors                  | Moravec’s Detector  
|                                 | Harris Detector     
|                                 | Scale Invariant Feature Transform 
|                                 | Affine Invariant Point Detector 
| Segmentation                    | Mean-Shift          
|                                 | Graph-Cut           
|                                 | Active Contours     
| Background Modeling             | Mixture of Gaussians  
|                                 | Eigenbackground     
|                                 | Wall Flower         
|                                 | Dynamic Texture Background 
| Supervised Classifiers          | Support Vector Machines  
|                                 | Neural Networks     
|                                 | Adaptive Boosting   |
Introduction – Object Tracking

- **Tracking**
  - Point Tracking (a)
  - Kernel Tracking (b)
  - Silhouette Tracking (c) & (d)
Introduction – Object Tracking

- Tracking Challenges
  - Correspondence
  - Occlusion

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**Fig.** Point correspondence. (a) All possible associations of a point (object) in frame $t-1$ with points (objects) in frame $t$, (b) unique set of associations plotted with bold lines, (c) multiframe correspondences.
Introduction – Vehicle Tracking

Vehicle Tracking

- **Motivation**
  - Traffic information
    - Reduce urban transportation industry costs
    - Future: Develop “intelligent” transportation system
  - Surveillance (I’d rather not mention)
    - Public Sector
    - Private Sector
Introduction — Vehicle Tracking

- **Object**
  - Track vehicles on a highway
  - Count them

- **Implementation**
  - Real-time
  - OpenCV & C++
Theory Overview

- **Segmentation**
  - Noise removal (minimization)
  - Background subtraction
  - Contour isolation
  - Rectangle fitting

- **Tracking**
  - Correspondence
  - Adding & removing vehicles
  - Persistence
  - Prediction
Theory – Segmentation

Noise minimization

- Gaussian Blur
  - Linear Convolution Filter
Theory – Segmentation

**Convolution**

\[ I_A(i, j) = I \ast A = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} A(h, k)I(i-h, j-k) \]

where \( A \) (and \( G \)) is the kernel and \( I \) is the image.

**Gaussian Kernel is Separable**

\[ I_G = I \ast G = \]

\[ = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} G(h, k)I(i-h, j-k) = \]

\[ = \sum_{h=-m/2}^{m/2} \sum_{k=-m/2}^{m/2} e^{\frac{h^2+k^2}{2\sigma^2}}I(i-h, j-k) = \]

\[ = \sum_{h=-m/2}^{m/2} e^{\frac{h^2}{2\sigma^2}} \sum_{k=-m/2}^{m/2} e^{\frac{k^2}{2\sigma^2}}I(i-h, j-k) \]

since \[ \frac{h^2+k^2}{2\sigma^2} = \frac{h^2}{2\sigma^2} + \frac{k^2}{2\sigma^2} \]
Theory – Segmentation

- **Gaussian Kernel is Separable**

Convoluting rows and then columns with a 1-D Gaussian kernel.

\[
\begin{align*}
I & \ast \frac{1}{38} \begin{bmatrix} 1 & 9 & 18 & 9 & 1 \end{bmatrix} = I_r \\
I_r & \ast \frac{1}{38} \begin{bmatrix} 1 \\ 9 \\ 18 \\ 9 \\ 1 \end{bmatrix} = \text{result}
\end{align*}
\]

The complexity increases linearly with \( m \) instead of with \( m^2 \).
Theory – Segmentation

Background Subtraction


- OpenCV implementation (without shadow detection)
Theory – Segmentation

Background Subtraction (continued)

- **Adaptive Gaussian Mixture Model**
  - Each pixel is modelled by a mixture of $K$ Gaussian distributions
  - $\text{BG Pixel} \leq T \text{ stdev}$
  - $\text{FG Pixel} > T \text{ stdev}$
  - where $T$ is the threshold
Theory — Segmentation

Background Subtraction (continued)

- **Online Expectation-Maximization (EM)**
  - Iterative parameter estimation
  - Benefits
  - Mathematica demonstration
Theory – Segmentation

- Finding Outside Contours
- Find Enclosed box
- Classification (simple for vehicles)
  - Keep boxes with size > Threshold
  (prevents noise from being detected as a car)
Correspondence

- Compare each new segmented object to each tracked object with the distance cost function:

\[ d^i = (a^i_x - b^i_x)^2 + (a^i_y - b^i_y)^2 \]

where \( a^i \) is the new object and \( b^i \) is the tracked object.

- Add each comparison that is less than \( T \) to a list.
- Order list (lowest cost first).
- Match first and remove all match with \( a^i \) and \( b^i \).
Theory – Vehicle Tracking

- **Adding Vehicles**
  - Mark all detected unmatched vehicles as potential
  - If found in next $g$ frames then add

- **Subtracting Vehicles**
  - All vehicles not found in $h$

- **Persistence**
  - Object not found within $h$ then not updated but still considered tracked

- **Occlusion**

$g, h \in [1, 2, ..., 10]$
Theory – Vehicle Tracking

**Prediction**

- **Kalman Filter**
  - Estimates a system’s state (optimal)
  - Maximizes a posteriori probability

- **Assumptions:**
  - System’s dynamics are linear
  - Noise is additive, white, and Gaussian
Kalman Filter (continued)

- **Current state vector** $x_k$
  \[ x_k = Fx_{k-1} + Bu_k + w_k \]

- $F$: transfer matrix
- $B$: relates the controls to $x_k$
- $u_k$: control vector
- $w_k$: the process noise vector
  - noise in state of the system.
  - $w_k$: random variable $N(0; Q_k)$. 
Kalman Filter (continued)

- **Measurement states** $z_k$
  
  $$z_k = H_k x_k + v_k$$

- $H_k$ : relates $x_k$ to $z_k$

- $v_k$ : measurement noise
  
  random variable with $N(0; R_k)$. 
Theory – Vehicle Tracking

Kalman Filter (continued)

- **Predict**
  
  \[
  x_k^- = Fx_{k-1} + Bu_{k-1} + w_k
  \]
  
  \[
  P_k^- = FP_{k-1}F^T + Q_{k-1}
  \]

- \(P_k\) : error covariance
Theory – Vehicle Tracking

Kalman Filter (continued)

- **Update**

\[
K_k = P_k^- H_k^- (H_k P_k^- H_k^T + R_k)^{-1}
\]

\[
x_k = x_k^- + K_k (z_k^- - H_k x_k^-)
\]

\[
P_k = (I - K_k H_k) P_k^{-}
\]

- \(K_k\): Kalman gain
  - weight to assign to new information
Theory – Vehicle Tracking

Kalman Filter Implementation Details

\[ x = \begin{bmatrix} x \\ y \\ v_x \\ v_y \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & dt & 0 \\ 0 & 1 & 0 & dt \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \]

\[ z = \begin{bmatrix} z_x \\ z_y \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \]
Results
Questions and Discussion